Optimal hedging relationships

Under FASB's statement 133, hedge accounting will be allowed only for designated pairs containing an item and matching derivative. Katherine Wyatt outlines a mathematical programme that companies can implement to choose an optimal hedge assignment that complies with the new reporting requirements

Statement 133, *Accounting for derivative instruments and hedging activities*, from the Financial Accounting Standards Board (FASB) requires that all derivative instruments be reported at fair value on the balance sheet, and hedge accounting will be allowed only for designated item/derivative pairs.

Choosing hedge relationships that satisfy the requirements for designated hedge pairs is now an important issue for companies that hold derivatives, and we present here a mathematical programme companies can implement to choose an optimal hedge assignment that complies with the new standards.

Statement 133 outlines the conditions under which the change in value of a derivative can be offset on the balance sheet by the change in value of a financial item, in an adjustment to the carrying amount of the hedged item. Derivative/item pairs that satisfy the hedge requirements must be designated at the start of the period and companies must demonstrate expected, as well as ongoing, hedge effectiveness. If this offset is perfect – that is, if the derivative exactly hedges the instrument – then the amount of gain or loss on the derivative that is reported in earnings is equal to zero.

These hedge accounting standards necessitate a method of determining the optimal item/derivative pairs and a method of demonstrating expected and current hedge effectiveness. Since it is common practice to institute hedges for total positions – which might, for example, be expressed as equivalents of five-year or 10-year US Treasuries – and because assets and liabilities are often basketed and hedged as portfolios, in many cases, entities may not be able to assume their hedges are effective.

A. Allowable risks for sample portfolio

Items	Allowable risks
Natural gas inventory	Market risk
Equity shares of XYZ	Market risk
Forecasted purchase of coffee	Market risk Foreign exchange risk
Corporate bond (US dollar) with quarterly coupon	Market risk Interest rate risk Credit risk
Deutschmark-denominated bond	Market risk Interest rate risk Foreign exchange risk

Statement 133 strongly suggests that items can be grouped together and hedged as a portfolio only if the percentage gain or loss on an individual item is within 1% or 2% of the percentage gain or loss on the whole portfolio over the period.

This means that items with a gain or loss not within these bounds for any portfolio the company holds can only be hedged individually. Further, the necessity of designating a hedged item/hedging derivative pair at the beginning of the period argues against reporting a dynamic hedging portfolio. Statement 133 notes that the hedge designation must be reported at the inception of the hedge. At that time, "there must be formal documentation of the hedging relationship and the entity's risk management objective and strategy for undertaking the hedge, including identification of the hedging instrument, the hedged transaction, the nature of the risk being hedged and details of how the hedging instrument's effectiveness in hedging the exposure to the hedged transaction's variability in cashflows attributable to the hedged risk will be assessed… Both at inception of the hedge and on an ongoing basis, the hedging relationship is expected to be highly effective in achieving offsetting cashflows attributable to the hedged risk during the hedge… An assessment of effectiveness is required whenever financial statements or earnings are reported, and at least every three months".

In November 1999, the derivatives implementation group of FASB reported that companies did not have to use the same methodology to assess effectiveness prospectively and retrospectively.

However, the FASB restrictions on hedging pairs can be expressed as logical requirements in a mathematical programme, and selecting optimal hedging designations can be modelled as an objective function which is the minimum of the total gain or loss on all derivative instruments that is not offset.

Assessing hedge effectiveness and assigning hedge relationships could involve front offices and back offices of a financial institution, as well as accounting, treasury and

B. Sample portfolio risk indicators

	Risk indicators
Market risk	S&P500 Index Natural gas six-month forward price Coffee six-month futures price
Market interest rate risk	US dollar Libor spot US dollar Libor six-month US dollar Libor 12-month Deutschmark Libor spot Deutschmark Libor six-month
Foreign exchange risk	US dollar/Brazilian Real exchange rate US dollar/Deutschmark exchange rate
Credit risk	Industry sector index

C. Gains or losses on items due to allowable risks

Item	Market risk	Interest rate risk	FX risk	Credit risk
Natural gas inventory	0.55(L)	0.0	0.0	0.0
Equity XYZ	0.21(G)	0.0	0.0	0.0
Forecasted purchase of coffee	0.31(L)	0.0	0.35(L)	0.0
US dollar debt	0.24(G)	0.46(G)	0.0	0.25(L)
German Bond (DM)	0.07(L)	0.24(L)	0.19(G)	0.0

G=gains L=losses

internal audit functions. However, managing hedge effectiveness fits most squarely as a function of risk management, as the effect of the new accounting standards is entity-wide and the risk management group in most organisations is already equipped to measure the sensitivities of the company's holdings to different market and credit factors.

A risk manager can use the programmes outlined below: first, to demonstrate the historical effectiveness of a proposed hedging relationship and second, to evaluate the performance of the hedge designations during a reporting period.

Hedge accounting criteria

The FASB statement distinguishes, first, between types of hedging relationships and, second, between risks (or attributable causes of change in value) that can be hedged against. Companies may elect to designate a derivative as a hedging instrument as follows:

☐ A hedge of the change in fair value of an asset or liability (fair value hedge);

☐ A hedge of the variability in cashflows of an asset or liability, an available-for-sale security, or of a forecasted transaction, or purchase or sale of an asset or liability (cashflow hedge); and

☐ A hedge of the foreign currency exposure of an asset or liability (foreign currency fair value hedge), an available-for-sale-security or a forecasted transaction (foreign currency cashflow hedge), or a net investment in a foreign operation.

The distinctions between fair value, cashflow and foreign currency hedges are reporting distinctions: the offset amount of gains and losses for derivatives are reported in earnings, or other accumulated income, or as a translation adjustment, depending on the type of hedge.

Gain or loss for a derivative in a fair value hedge is reported in earnings, and the gain or loss due to the hedged risk adjusts the carrying amount for the hedged item. The gain or loss for the derivative that is not offset remains in earnings.

In cashflow hedges, the effective part of the hedge (ie, the amount of gain or loss on the derivative that is offset by the gain or loss on the hedged item due to the hedged risk, and that offset) is reported in accumulated other income. This effective portion is transferred to earnings in the period where the cashflow or transaction occurs.

The ineffective part of the hedge is currently reported in earnings. Foreign currency fair value hedges follow the former fair value reporting rule, while foreign currency cashflow hedges follow the latter reporting rule.

The effective portion of the gain or loss on a financial instrument that is designated as an economic hedge of the net investment in a foreign operation is reported as a translation adjustment, ie, in a separate component of consolidated equity. The important point to remember, however, is that the ineffective part of the hedge (the amount of gain or loss that is not offset) is reported currently in earnings. The optimal hedge assignment will be the one that minimises this amount.

The statement next delineates the restrictions for certain items on the risks that can be hedged against. The four risks, or changes in value, that the statement allows hedging for are:

☐ Market risk: gains or losses attributed to changes in the market price of the entire hedged item;

☐ Market interest rate risk: gains or losses attributed to changes in market interest rates;

☐ Foreign exchange risk: gains or losses attributed to changes in foreign exchange risks; and

☐ Credit (default) risk: gains or losses attributed to changes in an obligor's creditworthiness.

According to the statement, there are restrictions for certain items on the risks that can be hedged against:

☐ Financial assets and liabilities, the variable cashflows of financial assets and liabilities, and the forecasted purchases and sales of financial assets and liabilities can be hedged against either market risk or market interest rate risk, foreign exchange risk or credit (default) risk. Two or more of the latter group can be hedged simultaneously if desired;

☐ All or a portion of held-to-maturity debt securities, and the cashflows related to a held-to-maturity security can be hedged against credit (default) risk only;

☐ Non-financial assets or liabilities can be hedged against market risk only; the forecasted purchase or sale of a non-financial asset or liability can be hedged against either market risk or foreign exchange risk;

☐ All or a portion of the prepayment option of a held-to-maturity debt security can be hedged against market risk only; and

☐ The foreign currency exposure of an unrecognised firm commitment or available-for-sale security can be hedged against foreign exchange risk. This is also true for foreign currency exposure to variability in the functional currency equivalent cashflows associated with a forecasted foreign currency denominated transaction (either intercompany or extra-entity). And, finally, the foreign currency exposure of a net investment in a foreign operation can be hedged against foreign currency gain or loss.

These restrictions have to be included in a programme that models maximising hedge effectiveness. See table A for possible allowable risks for a sample portfolio.

The first requirement under the statement is to demonstrate that the entity expects the gains or losses of the hedging derivative to be highly effective in offsetting the gains or losses in the hedged item.

In many institutions, the market risk

management group analyses and reports the sensitivity of the institution's holdings to a set of risk indicators and calculates the risks inherent in these exposures. One example of risk indicators is the RiskMetrics' documentation list of indexes and factors against which a portfolio can be analysed.

One approach to demonstrate that a company expects a derivative to be "effective" is to require that, in addition to having had offsetting gains or losses in the past, the hedged item and hedging derivative share sensitivity to at least one of the risk indicators.

The advantage of this approach is that companies already have this information about their holdings and it does not require intensive computer time as, for example, developing correlation matrices would. The shared sensitivities can be easily determined from standard risk reports.

Companies can determine how closely related the designated items and hedges should be by specifying how many risk indicators must be shared for a designation to qualify. The market risk management group is best situated – because of its study of market risk factors – to review the effectiveness of hedge designations, and to weigh the possible benefits from hedge ineffectiveness.

Table B presents risk indicators for the sample portfolio.

Formulation of hedge effectiveness

The elements of our model are:

The inputs of gains or losses for items, over the last financial period, broken down according to the four allowable risks. $lg_{k,a}$ = gain for item k due to risk a, and $ll_{k,a}$ = the loss for item k due to risk a. Table C shows the gains and losses for items in the sample portfolio; $lg_{1,1} = 0.55$ and $ll_{5,2} = 0.24$.

The inputs of gains or losses, over the last financial period, for the derivatives. Dg^j = gain from derivative j and Dl^j = loss from derivative j. Table D shows the gains and losses for derivatives in the sample portfolio; $Dg^3 = 0.17$ and $Dl^2 = 0.38$.

For convenience, we also specify

$$lgl_{k,a} = lg_{k,a} + ll_{k,a}$$

and

$$Dgl^j = Dg^j + Dl^j$$

The inputs of allowable risk sensitivities for the hedge designations:

$$q^j_{k,a} = 0$$

if derivative j and item k are both sensitive to an indicator for risk a, and

$$q^j_{k,a} = -1$$

if they do not share any sensitivity. For the sample portfolio,

$$q^1_{2,1} = q^4_{3,1} = q^2_{4,1} = q^3_{5,1} = q^2_{4,2} = q^3_{5,2} = q^5_{5,3} = 0$$

Variable

$$Y^j_{k,a}$$

for the change in value in item k due to allowable risk a that is hedged by derivative j.

Decision variable

$$Z^j_{k,a}$$

is a 0–1 variable that indicates whether the hedge of item k by derivative j against risk a is a permissible designation.

Decision variable W^j is a 0–1 variable that indicates whether derivative D^j can be designated as hedging any item.

The optimal hedge in our model is the one that is least ineffective, ie, the hedge which minimises the amount of gain or loss on a derivative which remains after offset. Therefore, the objective function for the model is the minimum of the sum over all derivatives of the absolute value of the change in value of the derivative minus any permissible offsetting change in an item.

There is no explicit recommendation for a particular measure of hedge effectiveness in FASB's statement 133. However, a traditional and simple test of hedge effectiveness is that the ratio of gains or losses on the derivative to the losses or gains on the hedged item be between 80% and 125% over the financial period. This test can be applied to historical returns to demonstrate an expectation that a proposed hedge will be effective.

Representing requirements

Hedge accounting requirements differ according to the risk being hedged, whether entire items and derivatives or portions of them are being used, and whether basis swaps or written options appear as hedging instruments. We will first consider the case where there are no basis swaps or written options in the set of hedging derivatives.

The gains and losses over the period should be easily extracted from available profit and loss and delta reports. Since we have excluded basis swaps from the set of derivatives, we assume here that at most, one of $\{lg_{k,a}, ll_{k,a}\}$ and $\{Dg^j, Dl^j\}$ is greater than 0, for all a, j, k. Assume an entity holds the sample portfolio of items and derivatives and has collected the input data for this portfolio.

D. Gains or losses on derivatives

Derivative	Gain	Loss
XYZ option	0.0	0.01
IRS US dollar	0.0	0.38
IRS Deutschmark	0.17	0.0
Coffee six-month futures	0.27	0.0
Deutschmark six-month forward	0.0	0.05

Hedges against market risk

Hedge designation is most straightforward when market risk is being hedged (since if market risk is hedged, it is the only allowable risk) and the items and derivatives are such that either the entire instrument or a percentage of it can be designated in a hedge. In this case, if we restrict our test of effectiveness to an upper bound on the item/derivative offset, we have a formulation with linear constraints and a linear objective function.

Constraints

To ensure that designated pairs have similar risk profiles, we require

$$\forall j \, \forall k \; q^j_{k,1} * Y^j_{k,1} \geq 0$$

If item k and derivative j are not sensitive to the same indicator, then

$$q^j_{k,1} = -1$$

and the constraint can be satisfied only by setting

$$Y^j_{k,1} = 0$$

Item k is hedged effectively by derivative j if the following inequalities hold, since these constraints require that the hedged gain for the item is less than 125% of the hedging loss on the derivative and the hedged loss on the item is less than 125% of the hedging gain on the derivative.

$$\forall j \sum_{k=1}^{m} lg_{k,1} * Y^j_{k,1} \leq 1.25 Dl^j$$

$$\forall j \sum_{k=1}^{m} ll_{k,1} * Y^j_{k,1} \leq 1.25 Dg^j$$

If $Dg^j = 0$ and if $ll_{k,1} > 0$, then

$$Y^j_{k,1}$$

is forced to 0. Similarly, if $Dl^j = 0$ and if $lg_{k,1} > 0$, then

$$Y^j_{k,1}$$

must be 0. This ensures that the gain or loss on a hedged item offsets the loss or gain on a hedging derivative.

Objective function: maximising hedge effectiveness is equivalent to minimising gain or loss from derivatives that is not offset:

$$\text{minimise: } \sum_{j=1}^{n} \left| Dgl^j - \sum_{k=1}^{m} lgl_{k,1} * Y_{k,1}^j \right|$$

Hedges against all four allowable risks: As noted above, if market risk is being hedged in fair value and cashflow hedges, then that is the only risk being hedged, and $a = 1$ for all j and k. However, if market risk is not being hedged, then one or more of interest rate risk, foreign exchange risk and credit risk can be hedged simultaneously in cashflow and fair value hedges. Therefore, to select the allowable designations that will provide the greatest offset a 0–1 variable

$$Z_{k,a}^j$$

is introduced, where $a \in 1, \ldots, 4$ and

$$Z_{k,a}^j = 1$$

$\Rightarrow$ derivative j and item change k,a are a designated hedge

$$Z_{k,a}^j = 0$$

$\Rightarrow$ derivative j and item change k,a *cannot* be a designated hedge

Then the requirement that a designated item and derivative are sensitive to the same indicators is expressed by

$$\forall a \forall j \forall k \; q_{k,a}^j * Z_{k,a}^j \geq 0$$

This is true because

$$Z_{k,a}^j$$

is either 0 or 1, and if

$$q_{k,a}^j = -1$$

indicating that item k and derivative j share no sensitivities, then the product

$$q_{k,a}^j * Z_{k,a}^j \geq 0$$

implies that

$$Z_{k,a}^j = 0$$

or that the hedge is not permitted.

Inequalities modelling the requirement that market risk not be hedged in conjunction with any other risk are added to the preceding programme:

$$\forall j \forall k \qquad Z_{k,1}^j + Z_{k,2}^j \leq 1$$

$$Z_{k,1}^j + Z_{k,3}^j \leq 1$$

$$Z_{k,1}^j + Z_{k,4}^j \leq 1$$

$$\forall a \forall j \forall k \qquad Y_{k,a}^j \leq Z_{k,a}^j$$

These inequalities are satisfied only if at most one of market risk and market interest rate risk, or market risk and foreign exchange risk, or market risk and credit risk is hedged. Then

$$Y_{k,a}^j$$

and

$$Z_{k,a}^j$$

are linked by requiring that if

$$Y_{k,a}^j = 1$$

then

$$Z_{k,a}^j$$

must also equal 1.

Constraints on the decision variables W^j can be used to enforce the lower bound for effectiveness:

$$\forall j \sum_{k,a} lgl_{k,a} * Y_{k,a}^j - 0.8 * Dgl^j * W^j \geq 0$$

Using this inequality forces $W^j = 0$ if all the

$$Y_{k,a}^j = 0$$

and if any

$$Y_{k,a}^j > 0$$

then the gain or loss on all the

$$Y_{k,a}^j$$

for that j must be at least 80% of the loss or gain on derivative j. The constraints modelling hedge effectiveness have to be expanded to include gains or losses due to all four allowable risks:

$$\forall j \sum_{a=1}^{4} \sum_{k=1}^{m} lg_{k,a} * Y_{k,a}^j \leq 1.25 Dl^j$$

$$\forall j \sum_{a=1}^{4} \sum_{k=1}^{m} ll_{k,a} * Y_{k,a}^j \leq 1.25 Dg^j$$

and the objective function is now:

$$\text{minimise: } \sum_{j=1}^{n} \left| Dgl^j - \sum_{a=1}^{4} \sum_{k=1}^{m} lgl_{k,a} * Y_{k,a}^j \right|$$

If we apply this formulation to the sample portfolio of items and derivatives, we find an optimal designation of:

- the loss from derivative 1 is offset by the gain from a portion (3.2%) of item 2;
- the loss from derivative 2 is offset by the gain due to interest rate risk of a portion (82%) of item 4;
- the gain from derivative 3 is offset by the loss due to interest rate risk of 70% of item 5;
- the gain from derivative 4 is offset by the loss from a portion (88%) of item 3; and
- the loss from derivative 5 is offset by the gain due to foreign exchange risk of a portion (29%) of item 5.

These designations yield perfect hedges: the offset for this example is 0.

Designating hedged pairs with entire items:

In the case of items of the following types, the entire item, not a portion, must be hedged:

(a) one or more selected contractual cashflows;

(b) a put option, a call option, an interest rate cap or an interest rate floor embedded in an existing asset or liability, and clearly and closely related to the host instrument;

(c) the residual value in a lessor's net investment in a direct-financing or sales-type lease.

For these items

$$Y_{k,a}^j$$

is replaced by the 0 – 1 variable

$$Z_{k,a}^j$$

in the constraints and the objective function.

Hedging with basis swaps or written options:

Using basis swaps and written options as hedging instruments carries additional restrictions. Basis swaps can be designated as hedging instruments only if they link an identified asset and liability, with the basis of one leg of the swap identical to the basis of the asset cashflows and the basis of the other leg identical to the basis of the liability cashflows. This requirement can be modelled by considering the basis swap as two derivatives; if the basis for each swap leg is included in the list of risk indicators for the portfolio, then we can require that the item hedged by each leg share sensitivity to the basis of that swap leg's cashflows.

A written option can only be designated as a hedge of an item with an embedded, closely related purchased option. A company can implement this requirement by including a "written option" indicator in the list of risk indicators, and specifying that an item will be recorded as sensitive to this indicator only if it has an embedded purchased option. A derivative will be sensitive to this indicator only if it is a written option.

Conclusions

We have presented a suite of mathematical programmes that risk managers can use to assign optimal hedge relationships among the items and derivatives held by a company. These programmes match items and derivatives so that the least amount of gain or loss on derivatives is reported while satisfying the requirements of FASB's statement 133. The programme for designating most items and derivatives in a hedge of market risk has a piecewise linear objective function and linear constraints, and can be solved using standard linear programming methods.

Programmes for designating all possible types of items and derivatives, in hedges of any allowable risk, include 0–1 decision variables and linear and disjunctive constraints. These latter programmes can be solved using branching search methods combined with linear programmes. Experiments with programmes modelling hedge effectiveness for portfolios of items and derivatives are being carried out at the Logic Based Systems Lab at Brooklyn College of the City University of New York. ■

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