Optimal Assignment of Financial Hedges in Satisfying Hedge Accounting Standards

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Abstract

A MIP implementation of a disjunctive model for optimal assignments of financial items and derivatives in hedge accounting is presented. Item and derivative sensitivity to risk factors is used to determine if certain hedges are permitted; disallowed hedges lead to elimination of rows and columns of the mixed integer program during preprocessing. Two "large" submatrices of the coefficient matrix for the MIP implementation, which have nonzero entries for the binary variables, are shown to be totally unimodular. Results from computer trials are reported: most of the large programs in the trials solved quickly, and in many the optimal MIP objective function value was equal to the optimal objective function value for its linear relaxation. Background information on the standards for hedge accounting is provided.

Key Words: optimization; finance; mixed-integer programming; accounting.

Introduction

Financial institutions often use derivatives to protect the earnings of their portfolios from changes in market factors. Companies take positions in derivatives that are expected to move opposite in value to financial items in their portfolios; these derivatives are called *hedges*. If a hedge relationship qualifies for hedge accounting, then an institution only reports in earnings the amount of gain or loss on the derivative (over the financial period) that

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remains after netting the gain or loss of the hedged item/hedging derivative pair.

New Financial Accounting Standards Board (*FASB*) rules for hedge accounting became effective for financial statements released after June 15, 2000. We present here a hedge-assignment model that uses disjunctive constraints to represent many of the criteria for hedge accounting. The outline of the paper is as follows: first, we provide background information on the accounting standards; next, we describe the hedge-assignment model and mixed-integer program; and finally, we discuss our implementation and present experimental results.

Background

Statement of Financial Accounting Standards No. 133, Accounting for Derivative Instruments and Hedging Activities, sets out the guidelines for the reporting and accounting of derivative instruments on a company's income statement and balance sheet.

FAS 133 sets the standards for *hedge accounting*, a method that nets the gain or loss from a derivative with the loss or gain of the item it is hedging. If hedge accounting is allowed, only the amount of gain or loss on a derivative that is not offset by loss or gain on the hedged item is reported on the financial statement. Companies can reduce the impact of on balance sheet accounting and potential volatility of reported income by using hedge accounting.

FAS 133 presents a framework for categorizing items and derivatives and the risks that can lead to changes in value over a financial period, as well as the requirements that must be satisfied for hedge accounting. First, derivative - item pairs that satisfy the requirements must be designated at the beginning of a financial period, so that gains and losses are netted for *previously* designated pairs. Second, companies must demonstrate, at the time of designation, that they expect the changes in value of the item - derivative pair to be offsetting, and that these changes are currently offsetting.

The requirements for implementing hedge accounting can be expressed as linear and disjunctive (here, $0-1$) constraints, and the problem of selecting optimal hedging designations can be modeled as minimizing the gain or loss from derivatives that is not offset over a financial period. An important feature of this formulation is that it uses data about a portfolio's sensitivity to market factors that is routinely collected and analyzed by risk management groups at financial institutions.

Hedge Accounting Criteria

The statement delineates the restrictions for certain items on the risks that can be hedged against. The four risks, or changes in value, that the statement allows hedging for are:

Market risk: gains or losses attributed to changes in the market price of the entire hedged item

Market interest rate risk: gains or losses attributed to changes in market interest rates

Foreign exchange risk: gains or losses attributed to changes in foreign exchange risks

Credit (default) risk: gains or losses attributed to changes in an obligor's creditworthiness

There are restrictions, however, on how financial items - assets and liabilities - are hedged against these risks. According to *FAS 133*,

> Financial assets and liabilities, the variable cash flows of financial assets and liabilities, and the forecasted purchases and sales of financial assets and liabilities can be hedged against either market risk, or market interest rate risk, foreign exchange risk, or credit (default) risk. Two or more of the latter group can be hedged simultaneously if desired.

This means that if a company chooses to assign derivatives to hedge the risk of a change in the market price of an item, then the company may not hedge the risk of a change in interest rates, or foreign exchange rates, or default probabilities as well.

The following restrictions also hold:

- All or a portion of held-to-maturity debt securities, and the cash flows related to to a held-to-maturity security, can be hedged against credit risk only.
- Nonfinancial assets or liabilities can be hedged against market risk only; the forecasted purchase or sale of nonfinancial assets or liabilities can be hedged against either market risk or foreign exchange risk.

- All or a portion of the prepayment option of a held-to-maturity debt security can be hedged against market risk only.
- In the case of items of the following types —one or more selected contractual cash flows; a put option, a call option, an interest rate cap, or an interest rate floor embedded in an existing asset or liability and clearly and closely related to the host instrument; and the residual value in a lessor's net investment in a direct-financing or sales-type lease —the entire item, not a portion must be hedged.
- Basis swaps can be designated as hedging instruments only if they link an identified asset and liability, with the basis of one leg of the swap identical to the basis of the asset cash flows, and the basis of the other leg identical to the basis of the liability cash flows.
- A written option can only be designated as a hedge of an item with an embedded, closely related purchased option.

Modeling Hedge Effectiveness

The first requirement under the statement is to demonstrate that the entity expects the gains or losses of the hedging derivative to be highly effective in offsetting the gains or losses in the hedged item.

In many institutions, the Market Risk Management group analyzes and reports the sensitivity of the institution's holdings to a set of risk indicators and calculates the risks inherent in these exposures. An example of risk indicators is the $RiskMetrics^{TM}$ documentation list of indices and factors against which a portfolio can be analyzed. One approach to demonstrating that a company expects a derivative to be "effective" is to require that, in addition to having had offsetting gains or losses in the past, the hedged item and hedging derivative share sensitivity to at least one of the risk indicators. The advantage of this approach is that companies already have this information about their holdings, and it does not require intensive computer time as, for example, developing correlation matrices would. The shared sensitivities can be easily determined from standard risk reports. Companies can determine how closely related the designated items and hedges should be by specifying how many risk indicators must be shared for a designation to qualify.

Model Formulation

In this model, a set of constants $\{q^j_{k,a}\}$ is used to represent whether derivative j and item k share sensitivity to risk a: if item k and derivative j are both sensitive to risk a, then $q^j_{k,a} = 0$. Imposing the constraint that allowable item/derivative pairs share sensitivity to the same risk factors supports the claim that hedged item/derivative pairs will have offsetting gains and risks in the future.

The model presented below includes hedging requirements for all items against the four major risks: market risk, interest rate risk, foreign exchange risk, and credit risk. We are assuming here that all items can be hedged in part, and that basis swaps and written options are not in the portfolio under consideration. Extensions to the model that can be used when items that must be hedged in entirety and basis swaps and written options are present in the portfolio are outlined in a later section of this paper.

The optimal hedge in our model is the one that is least ineffective, *i.e.*, the hedge which minimizes the amount of gain or loss on a derivative which remains after offset. Therefore, the objective function for the model is the sum over all derivatives of the absolute value of the difference of the change in value of each derivative minus any offsetting change in value in an item that it hedges.

There is no explicit recommendation for a particular measure of hedge effectiveness in *FAS 133*. However, a traditional and simple test of hedge effectiveness is that the ratio of gains or losses on the derivative to the losses or gains on the hedged item be between 80% and 125% over the financial period. This test can be applied to historical returns to demonstrate an expectation that a proposed hedge will be effective.

The elements of our model are

- the inputs of gains or losses for items, over the last financial period, broken down according to the four allowable risks. $Ig_{k,a}$ = gain for item k due to risk a, and $Il_{k,a}$ = the loss for item k due to risk a. $Ig_{k,a} = Il_{k,a} = 0$ if the standards do not allow item k to be hedged against risk a.

- the inputs of gains or losses, over the last financial period, for the derivatives. Dg^j = gain from derivative j and Dl^j = loss from derivative j.

- for convenience, we also specify $Igl_{k,a} = Ig_{k,a} + Il_{k,a}$ and $Dgl^j = Dg^j + Dl^j$. Only one of Dl^j or Dg^j can be greater than 0; only one of $Il^j_{k,a}$ of $Ig^j_{k,a}$ can be greater than 0.

- the constant risk sensitivities for the hedge designations: $q^j_{k,a} = 0$ if derivative j and item k are both sensitive to an indicator for risk a, and $q^j_{k,a} = -1$ if they do not share any sensitivity.

- variable $Y^j_{k,a}$ for the change in value in item k due to allowable risk a that is hedged by derivative j. $0 \leq Y^j_{k,a} \leq 1$.

- V^j is a variable for the difference between gain or loss on items hedged by derviative j and loss or gain on derivative j.

- decision variable $Z^j_{k,a}$ is a $0-1$ variable that indicates whether the hedge of item k by derivative j against risk a is a permissible designation.

- decision variable W^j is a $0-1$ variable that indicates whether derivative D^j can be designated as hedging any item.

Constraints

As noted above, if market risk is being hedged, then that is the only risk being hedged. However, if market risk is not being hedged, then one or more of interest rate risk, foreign exchange risk, and credit risk can be hedged simultaneously in cash flow and fair value hedges. To select the allowable designations that will provide the greatest offset a $0-1$ variable $Z^j_{k,a}$ is introduced, where $a \in 1, \ldots, 4$ and

$$\begin{aligned} Z^j_{k,a} = 1 \quad &\Rightarrow \quad \text{derivative } j \text{ and item change } k, a \text{ are a designated hedge} \\ Z^j_{k,a} = 0 \quad &\Rightarrow \quad \text{derivative } j \text{ and item change } k, a \text{ can } \textit{not} \text{ be a designated hedge} \end{aligned}$$

The requirement that a designated item and derivative are sensitive to the same indicators is expressed by

$$\forall a \ \ \forall j \ \ \forall k \ \ q^j_{k,a} * Y^j_{k,a} \geq 0$$

Inequalities modeling the requirement that market risk *not* be hedged in conjunction with any other risk are:

$$\begin{array}{llcl} \forall j \;\; \forall k & Z^j_{k,1} + Z^j_{k,2} & \leq & 1 \\ & Z^j_{k,1} + Z^j_{k,3} & \leq & 1 \\ & Z^j_{k,1} + Z^j_{k,4} & \leq & 1 \\ \forall a \;\; \forall j \;\; \forall k & Y^j_{k,a} - Z^j_{k,a} & \leq & 0 \end{array}$$

These inequalities are satisfied only if at most one of market risk and market interest rate risk, or market risk and foreign exchange risk, or market risk and credit risk is hedged. Then $Y^j_{k,a}$ and $Z^j_{k,a}$ are linked by requiring that if $Z^j_{k,a} = 0$, then $Y^j_{k,a}$ must also equal 0.

The constraints modeling hedge effectiveness include gains or losses due to all four allowable risks:

$$\forall j \;\; \sum_{a=1}^{4} \sum_{k=1}^{M} Ig_{k,a} * Y^j_{k,a} \leq 1.25 Dl^j$$

$$\forall j \;\; \sum_{a=1}^{4} \sum_{k=1}^{M} Il_{k,a} * Y^j_{k,a} \leq 1.25 Dg^j$$

Constraints on the decision variables W^j are used to enforce the lower bound for effectiveness:

$$\forall j \;\; \sum_{k,a} Igl_{k,a} * Y^j_{k,a} - .8 * Dgl^j * W^j \geq 0$$

Using this inequality forces $W^j = 0$ if all the $Y^j_{k,a} = 0$, and if any $Y^j_{k,a} > 0$ then the gain or loss on all the $Y^j_{k,a}$ for that j must be at least 80% of the loss or gain on derivative j. The decision variable $Z^j_{k,a}$ is then linked to W^j:

$$\forall j \;\; \sum_{k=1}^{M} \sum_{a=1}^{4} Z^j_{k,a} - 4M * W^j \leq 0$$

Each item is only hedged once across all risks:

$$\forall k \;\; \sum_{j=1}^{N} \sum_{a=1}^{4} Y^j_{k,a} \leq 1$$

The following inequalities constrain V^j to be less than or equal to the absolute value of the difference between the change in value for a derivative and the changes in value for items the derivative is hedging.

$$\forall j \;\; (\sum_{k=1}^{M}\sum_{a=1}^{4} Igl_{k,a} * Y^j_{k,a}) - V^j \leq Dgl^j$$

$$\forall j \;\; -(\sum_{k=1}^{M}\sum_{a=1}^{4} Igl_{k,a} * Y^j_{k,a}) - V^j \leq -Dgl^j$$

The objective function is:

$$\text{maximize} \sum_{j=1}^{N} -V^j$$

Extensions of Model

As noted earlier, for some items the hedge must be for the entire item, not a portion of it. If these items, *e.g.,* put or call options, are present, then the variable $Y^j_{k,a}$ for these k is replaced in the constraints by the $0-1$ variable $Z^j_{k,a}$. If the portfolio under consideration contains a basis swap, then the hedging requirements can be modeled by treating the basis swap as two derivatives and by including the basis for each leg in the list of risk indicators for the portfolio. Then we can require that the items designated as being hedged by either leg of the swap are sensitive to the relevant basis indicator for the swap leg. A company can implement the requirement that a written option can only be designated as a hedge of an item with an embedded closely related purchased option by including a "written option indicator" in the list of portfolio risk indicators. An item can then be recorded as sensitive to this indicator only if it has an embedded purchased option, and a derivative will be sensitive to this indicator only if it is a written option.

Model Implementation

In an implementation of the model at the Logic Based Systems Lab at Brooklyn College, a set of constants $\{q^j_{k,a}\}$ was used to code whether derivative j and item k shared sensitivity to risk a. These constants depended on risk sensitivities that were generated randomly for each item and derivative: if the random number generated was greater than a threshold value, then the particular item or derivative was "sensitive" to that particular risk. This threshold can be interpreted as the probability that an item and derivative will not share sensitivity to a particular risk indicator. Every item/derivative pair was tested to see if both were sensitive to the same risk indicator: if item k and derivative j shared sensitivity to the same indicator for a certain risk a then $q^j_{k,a}$ is set to 0; if item k and derivative j do not share sensitivity to any indicator for risk a then $q^j_{k,a}$ is set to -1. Users could also require that item/derivative pairs were sensitive to more than one indicator for a particular risk to determine shared sensitivity to the risk.

In the experimental trials reported on below, numbers representing the change in value of the items that were due to each risk were generated randomly.

Coefficient Matrix

It is convenient to consider the submatrices of the coefficient matrix of this implementation. These submatrices can be represented as below, where *i, ii, iii*, and *iv* refer to column sets and A through L refer to coefficient submatrices, before preprocessing has taken place.

Columns

i: set of $4MN$ continuous variables $Y^j_{k,a}$

ii: set of $4MN$ binary variables $Z^j_{k,a}$

iii: set of N binary variables W^j

iv: set of N continuous variables V^j

Submatrices

$$\begin{array}{cccc} i & ii & iii & iv \end{array}$$

$$\begin{pmatrix} A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ C & D & 0 & 0 \\ E & 0 & F & 0 \\ 0 & G & H & 0 \\ I & 0 & 0 & 0 \\ J & 0 & 0 & 0 \\ K & 0 & 0 & L \end{pmatrix}$$

A: $4MN \times 4MN$ diagonal submatrix of shared risk sensitivities

B: $3MN^2 \times 4MN$ submatrix that models requirement that market risk must be hedged alone

C: $4MN \times 4MN$ diagonal submatrix with entries of $+1$

D: $4MN \times 4MN$ diagonal submatrix with entries of -1

C and D together enforce requirements for allowed hedges

E, F, G, H: $N \times 4MN$, $N \times N$, $N \times 4MN$, and $N \times N$ submatrices that express lower bound constraints

I: $2N \times 4MN$ submatrix that expresses the upper bound constraints

J: $M \times 4MN$ submatrix that ensures that each item is hedged only once

K, L: $2N \times 4MN$ and $2N \times N$ submatrices model the absolute value of derivative gain or loss not offset by hedging

Total Unimodularity of Submatrices

Note that submatrices B, D, and G represent the coefficients for the binary variables $Z^j_{k,a}$ and submatrices F and H represent the nonzero coefficients of binary variables W^j. Consider the coefficient submatrix B that represents the requirement that if the market risk of an item is hedged, then interest rate risk, foreign exchange risk, and credit risk cannot be hedged against for this item. B has $3MN^2$ rows and $4MN$ columns and each row refers to a constraint of the form

$$Z^j_{k,1} + Z^{j\prime}_{k,a} \leq 1$$

where $j, j\prime \in \{1, \ldots N\}$ and $a \in \{2, 3, 4\}$.

Lemma 1 *Every collection of columns of* B *can be split into two parts so that the sum of the columns in one part minus the sum of the columns in the other part is a vector with entries of only* $0, +1,$ *or* -1.

Proof. Every row of B has exactly two nonzero entries, each equal to $+1$. If a collection of columns contains columns exclusively from the set $\{Z^j_{k,a} | a = 1\}$ or exclusively from the set $\{Z^j_{k,a} | a \in \{2, 3, 4\}\}$ then for every partition of the collection into two parts, the sum of the columns in one part minus the sum of the columns in the second part will yield a vector with entries of $0, +1$, or -1 since there is at most one $+1$ in each row in each collection. If a collection of columns contains columns from both sets $\{Z^j_{k,a} | a = 1\}$ and $\{Z^j_{k,a} | a \in \{2, 3, 4\}\}$, it is possible to form a partition according to these sets. Then the sum of the columns in one part minus the sum of the columns in the second will yield a vector with entries $0, +1$, or -1 since there will be at most one nonzero entry of $+1$ in each row in each partition.

Claim 1 *B is totally unimodular.*

Proof. The Claim follows from Lemma 1 and from Ghouli-Houri (1962), as reported in Schrijver (1986).

Claim 2 *D is totally unimodular.*

Proof. D is a diagonal matrix with entries of -1 on the diagonal, so by definition D is totally unimodular.

As noted earlier, B is a $3MN^2 \times 4MN$ submatrix and D is $4MN \times 4MN$ diagonal submatrix, and F, G, and H are the only other submatrices of the coefficient matrix with nonzero coefficients for the binary variables $Z^j_{k,a}$ and W^j. Since F, G, and H are of size $N \times N$, $N \times 4MN$, and $N \times N$ respectively, then the larger part of the submatrix of nonzero coefficients for the binary variables is totally unimodular.

Elimination of Rows and Columns in Preprocessing

As noted above, A is a diagonal submatrix of the $q^j_{k,a}$, *i.e.* with 0 or -1 on the diagonal. Recall that $q^j_{k,a} = 0$ if item k and derivative j share sensitivity to risk a, and $q^j_{k,a} = -1$ if they do not share sensitivity to risk a. Preprocessing (*i.e.* the presolve function in CPLEX) was used with the constraints that this coefficient submatrix represents to "prune" item/derivative pairs from the program if they were not sensitive to the same risk indicators. If $q^j_{k,a} = -1$, then the column corresponding to $Y^j_{k,a}$ was eliminated since the constraint forces $Y^j_{k,a} = 0$. In fact, the entire $4MN \times 4MN$ submatrix A was eliminated since, in addition to the column eliminations, the $a * j + k$th row was eliminated if $q^j_{k,a} = 0$.

Experimental Results

Experimental trials of this model were run at the Logic Based Systems Lab at Brooklyn College of the City University of New York. The optimization problem was coded in CPLEX 5.0 and trials were run on a Sun Ultra Enterprise 4000.

As mentioned above, a threshold value was used with a random number generator to determine if an item or derivative was sensitive to a particular risk indicator: a random number was generated for each item and derivative and risk indicator, and if the random number was greater or equal to the threshold, then the item or derivative was said to be sensitive to that risk indicator.

Three different sensitivity thresholds were used in test sets, where each consisted of 100 trial runs of the CPLEX program. The sensitivity threshold for Test Set 1 was .60, for Test Set 2 was .75, and for Test Set 3 was .85. As the sensitivity threshold got higher, fewer items and derivatives shared sensitivity to the same risk indicators, and, therefore, fewer columns remained for the same initial size programs as the threshold increased. The initial size of the program (before preprocessing) depended on M, the number of items, and N, the number of derivatives. The initial coefficient matrix has $8MN + 2N$ columns and $8MN + 3MN^2 + 6N + M$ rows.

Run Times

In each of the three tests reported on here, M and N ranged between 30 and 39; a program was run for each of the 100 combinations of these ten numbers. The number of rows, columns, and nonzero entries (before preprocessing) for the case when $M = N = 39$ was $190,398$ rows, $12,246$ columns, and $416,910$ nonzero entries. In contrast, the number of rows, columns, and nonzero entries remaining for this case after preprocessing was $24,056$ rows, $3,850$ columns, and $59,116$ nonzeros when the sensitivity threshold was set at .60; $9,235$ rows, $2,172$ columns, and $24,456$ nonzeros when the threshold was .75; $1,487$ rows, 679 columns, and $4,586$ nonzeros when the threshold was .85.

Machine times for the CPLEX presolve phase and for program execution were recorded for each of the 300 computer runs in the three tests. A time limit of two hours was set, and if no solution had been found at the end of two hours, the program ended and returned a status indicator of time limit exceeded. The linear relaxation of each mixed integer program that remained after preprocessing was also solved. Times for programs that failed to finish within the time limit were excluded from analysis.

It is interesting to note that the number of programs that failed to complete within two hours increased as the threshold increased: all the programs completed within two hours when the threshold was .60; two programs failed to complete in two hours when the threshold was .75; and five programs failed to complete within two hours when the threshold was .85.

The average time for the presolve phase was 3.49 seconds for Test Set 1, with actual times for presolve ranging between 2.2 seconds and 5.25 seconds; the average time to solution for the mixed integer programs in Test Set 1 was 61.34 seconds, with a minimum time of 24 seconds and a maximum time of 158 seconds. The average time for solution of the linear relaxations of the MIPs in Test Set 1 was 5.15 seconds, with actual values between 2.86 seconds and 9.41 seconds.

Test Set 2 times - when the times for the two programs that failed to complete within two hours were removed from consideration – were shorter. Average presolve time was 3.06 seconds, with a minimum time of 1.89 seconds and a maximum time of 4.7 seconds; the MIPs were solved on average in 11.05 seconds, with the fastest time to solve 4 seconds and the slowest program solved in 28 seconds. All the linear relaxations of the MIPs in Test Set 2, including the relaxations of the two programs that failed to finish in two hours, were solved within the time limit. The average time for the linear relaxations of the MIPs in Test Set 2 was 3.67 seconds, with a minimum of 2.33 seconds and a maximum of 5.8 seconds.

Test Set 3 programs had the shortest solution times for the majority of the trials – here the MIP was solved in less than 10 seconds for 89 of 100 programs. However, in addition to the five programs that failed to finish within 2 hours, there were two programs that took 1849 and 3776 seconds respectively. On average the presolve phase took 2.8 seconds and the linear relaxations of the MIPs took on average 3.4 seconds, with a minimum time of 2.08 seconds and a maximum time of 4.94 seconds.

Objective Function Values

The differences in times recorded for the different trials is related to the number of rows, columns, and nonzeros that remain after preprocessing with the three different threshold values. The results are also different for the three Test Sets when one counts two other totals: the number of programs in which the optimal objective function value for the MIP equals the optimal objective function value of the MIP's linear relaxation; and the number of programs in which the optimal objective function value for the MIP and the linear relaxation is equal to 0, indicating the solution is a perfect hedge. In the first Test Set, where the threshold was set at .60, all the MIPs achieved the optimal objective function value of their linear relaxations and 92 of the solutions were perfect hedges. In Test Set 2, where the threshold was set at .75, 89 of the MIPs had optimal objective function values equal to the optimal objective function values of the MIP's linear relaxations, and only 26 of these were perfect hedges. For Test Set 3, with a threshold of .85, the number of programs where the optimal objective function value for the MIP equalled the optimal objective function value of the linear relaxation was 61, but no program had a perfect hedge solution.

Conclusions

Companies can use the mixed-integer program presented here to assign optimal hedge relationships among the items and derivative held in their portfolios. Standard risk management reports can be used in modeling items' and derivatives' sensitivity to different risk factors. The program matches items and derivatives so that the least amount of gain or loss on derivatives is reported and hedge accounting standards are satisfied.

Results of experiments carried out at the Logic Based Systems Lab at Brooklyn College of the City University of New York showed that the "sensitivity threshold" used in the program influenced the number of rows, columns, and nonzeros eliminated during preprocessing. The number of programs in each test set in which the MIP optimum equals the linear relaxation optimum seems also to be related to these threshold values. One interpretation of these results is that if there are more allowed hedges (as in Test Set 1) then it is more likely that the linear relaxation optimum will be achieved by the MIP, and more likely that a perfect hedge is possible. Alternatively, when the likelihood of allowed hedges is reduced, as in Test Sets 2 and 3, then it is less likely also that the MIPs will achieve the linear relaxation optimum or that there will be perfect hedges. Users can exploit this behavior by grouping the items and derivatives for programs so that the likelihood that an item or derivative is sensitive to a particular risk factor is not set too low.

In the vast majority of the trials, the MIP optimum equalled the linear relaxation optimum. This result was most likely due in part to the total unimodularity of the larger part of the submatrix with nonzero coefficients for the binary variables. Also in the vast majority of MIP trials, programs solved quickly, and this indicates that the model may be tractable in finding optimal hedge assignments for many large portfolios.

	Sensitivity Limit = 0.60 100 *Trials*	*Sensitivity Limit = 0.75* 100 *Trials*	*Sensitivity Limit = 0.85* 100 *Trials*
Machine Times	*sec.*	*sec.*	*sec.*
Presolve			
minimum	2.2	1.9	1.71
maximum	5.25	4.7	4.44
average	3.5	3.1	2.9
MIP Optimization			
minimum	24	4	2
maximum	158	28	1849
average	61.3	11.1	23.9
Linear Relaxation			
minimum	2.9	2.33	2.03
maximum	9.4	5.8	4.9
average	5.1	3.7	3.4
% Left After Presolve			
% Rows			
minimum	9.0	2.6	0.5
maximum	15.4	5.5	1.8
average	12.2	4.0	1.1
% Columns			
minimum	25.9	12.0	4.2
maximum	35.1	19.6	9.1
average	30.7	15.8	6.8
% Nonzeros			
minimum	10.5	3.4	0.7
maximum	17.0	6.7	2.4
average	13.8	5.0	1.5
Failed to solve within 2 hours	0	2	5
MIP optimum = LR optimum	100	89	61
Perfect hedges	92	26	0

Number of items and derivatives ranged between 30 and 39. Programs which did not finish within 2 hours are not included in analysis.

Selected Readings

Katherine Wyatt, "Optimal Hedging Relationships," Special Report on Financial Accounting Standard 133, *Risk* Magazine, May 2000.

Financial Accounting Standards Board, *Statement of Financial Accounting Standards No. 133*, Financial Accounting Series, No. 186-B, June 1998.

J.P. Morgan/Reuters, *RiskMetrics*^{TM} Technical Document, 4th Edition, Morgan Guaranty Trust Company of New York, 1996.

Alexander Schrijver, *Theory of Linear and Integer Programming*, Wiley, Chichester, 1986.

H.P. Williams, *Model Building in Mathematical Programming*, 3rd Ed., John Wiley & Sons, Chichester, 1990.

Katherine Wyatt, *Decomposition Techniques and Disjunctive Linear Programming for Fixed-Income Portfolio Selection*, doctoral dissertation, Mathematics, City University of New York (1997).