Discrete Structures - Final exam

Name:____________

Answer all six questions.

Question 1 [10 points]

Consider the relation R on $S = \{a, b, c, d\}$ defined as

$$R = \{(a, a), (a, b), (b, a), (b, b), (c, c), (c, d), (d, c), (d, d)\}$$

(1) Which of the properties (reflexive, anti-reflexive, symmetric, anti-symmetric, and transitive) does R satisfy?

(2) Is R a function from S to S? Why?

(3) Is R an equivalence relation? Why?

(4) Give an adjacency matrix to represent the relation R.

(5) Draw a digraph to represent the *reachability relation* of R.

Question 2 [6 points]

Circle all the propositions in the following that are tautologies.

(1) $\neg q \to (q \to p)$

(2) $(\neg p \vee q) \leftrightarrow (p \to q)$

(3) $p \wedge \neg p$

(4) $(p \vee q) \wedge (\neg p \to q)$

(5) $\neg p \to \neg q$

(6) $\neg(\neg p \vee q) \leftrightarrow (p \to q)$

Question 3 [6 points]

Prove that the product of x, $x + 4$, and $x + 8$ is divisible by 3 for all $x \in Z$.

Question 4 [6 points]

Consider the recurrence relation $s_n = 5s_{n-1} - 4s_{n-2}$, where $s_0 = 2, and s_1 = 5$. Prove by induction that

$$s_n = 4^n + 1$$

for all $n \in N$.

Question 5 [4 points]

Define a recursive function $f(n)$ for each of the following sequences:

(a) $\frac{1}{2},\ \frac{1}{4},\ \ldots,\ \frac{1}{2^n},\ \ldots$

(b) $1^2,\ 1^2 \times 2^2,\ \ldots,\ \prod_{i=1}^{n} i^2,\ \ldots$

Question 6 [8 points]

Let Σ be the alphabet {a,b,c,d} and $\Sigma^k = \{w \in \Sigma^* : length(w) = k\}$. How many elements are there in each of the following sets?

(a) $\{w \in \Sigma^4 :$ no letter in w is used more than once.$\}$

(b) $\{w \in \Sigma^4 : w$ begins with letter `a` or letter `b`.$\}$

(c) $\{w \in \Sigma^4 :$ the letter `a` occurs in w at least once.$\}$

(d) $\{\{w_1, w_2\} : w_1, w_2 \in \Sigma^4$, one word does not contain letter `a` and the other word does not contain letter `b`.$\}$