

# Discrete Structures - Test 2

Name: \_\_\_\_\_

Answer all five questions.

## Question 1 [6 points]

Consider the relation on  $S = \{0, 1, 2, 3\}$  defined as  $R = \{(m, n) | m \in S, n \in S, m \leq n\}$ .

- (a) Write  $R$  as a set of ordered pairs.
- (b) Which of the properties (reflexive, anti-reflexive, symmetric, anti-symmetric, and transitive) does  $R$  satisfy?
- (c) Draw the digraph for  $R$ .
- (d) Draw a matrix for  $R$ .

## Question 2 [3 points]

Consider the relation  $R$  on  $\{1, 2, 3, 4\}$  defined as

$$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$$

Is  $R$  an equivalence relation? If yes, what are the equivalent classes [1] and [2]? If no, explain.

## Question 3 [3 points]

Prove by induction that

$$1 \times 2 + 2 \times 3 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

for all  $n \in P$ .

## Question 4 [4 points]

Consider the recursively defined sequence  $S$  defined as

$$\begin{aligned} \text{(B)} \quad & S(1) = 1, S(2) = 1 \\ \text{(R)} \quad & S(n) = S(n-1) + S(n-2) \end{aligned}$$

Calculate  $S(9)$  and  $S(11)$ .

## Question 5 [4 points]

Give a recursive definition for each of the following sequences:

- (a)  $1!, 2!, 3!, \dots$
- (b)  $5^1, 5^2, 5^3, \dots$