Programming Languages
Sample Final Exam

Question 1
Give a regular expression for each of the following languages over $\Sigma = \{0, 1, \ldots, 9\}$.
1. All 5-digit integers that contain no leading zeros.
2. All positive integers that begin with 9 and that are multiples of 5.
3. All strings that begin with 9 and contain three consecutive 1s.

Question 2
Give a DFA for each of the languages in Question 1.

Question 3
Give a context-free grammar for each of the following languages over $\Sigma = \{a, b\}$.
1. $a^*b^*$
2. Strings that contain the same number of $a$’s as $b$’s.
3. $\{a^n b^n + k | 0 \leq k\}$

Question 4
Consider the following grammar.
$$E \rightarrow E \text{ or } E \mid E \text{ and } E \mid \text{not } E \mid (E) \mid x$$
1. Prove that the grammar is ambiguous by giving an example sentence for which there are two or more parse trees.
2. Assume that the operators $\text{or}$ and $\text{and}$ are left-associative, the operator $\text{not}$ is right associative, and that the operators have the precedence relation: $\text{not} > \text{and} > \text{or}$. Rewrite the grammar into one that does not have ambiguity and respects the associativity and the precedence of the operators.

Question 5
Write the following functions in Picat, Haskell, or Python using recursion. No higher-order functions or list comprehensions can be used in the implementations.
1. $\text{my_zip}(\text{lst1}, \text{lst2})$: Let $\text{lst1}$ be $[A_1, A_2, \ldots, A_n]$, and $\text{lst2}$ be $[B_1, B_2, \ldots, B_n]$. This function returns the association list $[\langle A_1, B_1 \rangle, \langle A_2, B_2 \rangle, \ldots, \langle A_n, B_n \rangle]$.
2. $\text{lookup}($alist$, x)$: This function returns the value associated with $x$ in the association list $\text{alist}$. For example,
$$\text{lookup}(\langle [\langle \text{‘a’, 1}, \langle \text{‘b’, 2}, \langle \text{‘c’, 3}\rangle\rangle \rangle, \text{’b’}\rangle)$$
returns 2.
3. $\text{replicate}($lst$, n)$: This function replicates the elements of $\text{lst}$ $n$ times. For example
$$\text{replicate}([\langle \text{‘a’, ‘b’, ‘c‘}\rangle, 3)$$
returns ['a', 'a', 'a', 'b', 'b', 'b', 'c', 'c', 'c'].

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Question 6

Design a data structure for binary trees, and write the following functions on binary trees in Picat, Haskell, or Python.

1. `leaves(tree)`: This function returns a list of leave values in `tree` from left to right.

2. `deepest(tree)`: This function returns the values in a deepest node in `tree`. If there are multiple such values, then the functions returns the left-most one.

3. `min_max(tree)`: This function returns a pair `(min, max)`, where `min` is the minimum element, and `max` is the maximum element in `tree`. Note that the tree may not be a binary search tree.