Programming Languages and Compilers
Midterm Exam

Complete the exam, and submit it as a plain text email under the subject "CISC 7120 Midterm" to nzhou@brooklyn.cuny.edu by midnight on Wednesday, October 20.

Question 1

Consider ternary (base-3) numbers over the alphabet \( \Sigma = \{0, 1, 2\} \). Give a regular expression for each of the following languages:

1. All 3-digit numbers with no repeating digits. For example, "012" is valid, but "12" and "100" is not.

2. All numbers in which the digits are increasing. For example, "02" is valid, but "0210" is not.

3. All numbers with at least two repeating digits. For example, 0101 and 1122 are valid, but 0 and 020 are invalid.
Question 2

The following DFAs define three languages over $\Sigma = \{a, b\}$. Describe each of the languages, and give a regular expression for it.

Figure 1: (DFA-1)

Figure 2: (DFA-2)

Figure 3: (DFA-3)
Question 3

Write a function in a programming language of your choice that takes a string and returns true if and only if the string is accepted by DFA-3 given above.
Question 4

Give a context-free grammar for each of the following languages over $\Sigma = \{a, b, c\}$.

1. $\{a^n b^{2n} : n \geq 0\}$.

2. Strings of a’s and b’s with twice as many a’s as b’s.

3. $\{a^n b^m c^k : k = n + m\}$
Consider the following grammar.

\[
\begin{align*}
E & \rightarrow E \circ T \mid T \\
T & \rightarrow T \cdot NF \mid NF \\
NF & \rightarrow n \cdot NF \mid F \\
F & \rightarrow (E) \mid t \mid f
\end{align*}
\]

where \(E, T, NF, \) and \(F\) are non-terminals, and \(\circ, \cdot, n, (, , ) , t, \) and \(f\) are terminals.

1. Give a leftmost derivation for the following expression.

\[
n \cdot (t \cdot (n \cdot f)) \cdot a \cdot t
\]

2. Describe the associability (left-associative or right-associative) of the operators: \(\circ, \cdot, \) and \(n.\)

3. Eliminate left-recursion from the grammar.

4. (extra 10 points) Write a top-down recursive descent parser in a programming language of your choice for the grammar obtained after the above transformation.