Question 1

Give a regular expression for each of the following languages over $\Sigma = \{0, 1, \ldots, 9\}$.

1. All strings that contain no leading zeros (e.g., 00 and 012 are invalid, but 0 and 12 are valid).
2. All strings that represent even numbers (strings with leading zeros are allowed).
3. All strings that contain three consecutive 1’s (strings with leading zeros are allowed).

Question 2

Give a DFA for each of the languages in Question 1.

Question 3

Give a context-free grammar for each of the following languages over $\Sigma = \{a, b\}$.

1. $n$ a’s followed by $m$ b’s, where $n \leq m + 3$.
2. The number of a’s is different from the number of b’s.

Question 4

Consider the following grammar.

```
E -> E or T | T  
T -> T and NF | NF  
NF -> not NF | F  
F -> ( E ) | x
```

Construct a parse tree for the sentence:

```
not not x or not x and x
```

Question 5

Define the following functions in Python.

1. `assoc(lst1, lst2)`: Let $lst1$ be $[A_1, A_2, \ldots, A_n]$, and $lst2$ be $[B_1, B_2, \ldots, B_n]$. This function returns the association list $[(A_1, B_1), (A_2, B_2), \ldots, (A_n, B_n)]$.
2. `ass1(alist, x)`: This function returns the value associated with $x$ in the association list $alist$. For example,
   
   ```python
   ass1([('a', 1), ('b', 2), ('c', 3)], 'b')
   ```
   returns 2.

3. `repli(lst, n)`: This function replicates the elements of $lst$ $n$ times. For example
   
   ```python
   repli(['a', 'b', 'c'], 3)
   ```
   returns ['a', 'a', 'a', 'b', 'b', 'b', 'c', 'c', 'c'].
