

## Homework Solutions - Section 1.4

1.

(a)  $A \cup B = \{1, 2, 3, 5, 7, 9, 11\}$

(b)  $A \cap C = \{3\}$

(c)  $C^c = \{1, 4, 5, 7, 8, 9, 10, 11\}$

$(A \cup B) \cap C^c = \{1, 5, 7, 9, 11\}$

(d)  $A \setminus B = \{1, 9\}$

(e)  $C \setminus D = \{3, 6, 12\}$

(f)  $B \oplus D = \{3, 4, 5, 7, 8, 11\}$

(g) 16

3.

(a)  $[2, 3]$

(b)  $[0, 6]$

(c)  $[0, 2)$

(d)  $[0, 2) \cup (3, 6]$

(e)  $(-\infty, 0) \cup (3, \infty)$

(f)  $\emptyset$

(g)  $\mathbb{N}$

(h)  $[0, 2]$

(i)  $\emptyset$

5.

$$A = \{a, b, aa, bb, aaa, bbb\}$$

$$B = \{w \in \Sigma^* : \text{length}(w) \geq 2\}$$

$$C = \{w \in \Sigma^* : \text{length}(w) \leq 2\} = \{\lambda, a, b, aa, ab, ba, bb\}$$

$$A^c = \Sigma^* \setminus A$$

$$B^c = \{w \in \Sigma^* : \text{length}(w) < 2\} = \{\lambda, a, b\}$$

$$C^c = \{w \in \Sigma^* : \text{length}(w) > 2\}$$

$$(a) B^c \cap C^c = \emptyset$$

$$(b) B \cap C = \{w \in \Sigma^* : \text{length}(w) = 2\} = \{aa, ab, ba, bb\}$$

$$(B \cap C)^c = \{w \in \Sigma^* : \text{length}(w) \neq 2\}$$

(c) same as part (a) by DeMorgans Law

(d) same as part (b) by DeMorgans Law

$$(e) A^c \cap C = \{\lambda, ab, ba\}$$

$$(f) A^c \cap B^c = \{\lambda\}$$

7.

$$A \oplus A = \emptyset$$

$$A \oplus \emptyset = A$$

11.

$$A = \{a, b, c\}$$

$$B = \{a, b, d\}$$

$$(a) A \times A = \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c)\}$$

$$(b) A \times B = \{(a,a), (a,b), (a,d), (b,a), (b,b), (b,d), (c,a), (c,b), (c,d)\}$$

$$(c) \{(x,y) \in A \times B : x = y\} = \{(a,a), (b,b)\}$$

13.

$$(a) \{(0,0), (1,1), (2,2), (3,3), (4,4), (5,5), (6,6), \dots\}$$

$$(b) \{(0,2), (1,1), (2,0), (3,0), (0,3), (0,5), (5,0), \dots\}$$

$$(c) \{(6,1), (6,2), (6,3), (6,4), (6,5), (6,6), (6,7), \dots\}$$

$$(d) \{(3,4), (3,5), (3,6), (3,7), (3,8), (3,9), (3,10), \dots\}$$

$$(e) \{(3,1), (3,2), (3,3), (1,3), (2,3)\}$$

$$(f) \{(0,0), (1,1), (2,4), (3,9), (4,16), (5,25), (6,36), \dots\}$$