

Homework Solutions - Section 1.6

1.

(a) $7!/5! = 7 \cdot 6 = 42$

(b) $10!/6!4! = 10 \cdot 9 \cdot 8 \cdot 7/4! = 210$

(c) $9!/0!9! = 1$

(d) $8!/4! = 8 \cdot 7 \cdot 6 \cdot 5 = 1680$

(e)

$$\sum_{k=0}^5 k! = 0! + 1! + 2! + 3! + 4! + 5! = 1 + 1 + 2 + 6 + 24 + 120 = 154$$

(f)

$$\prod_{j=3}^6 j = 3 \cdot 4 \cdot 5 \cdot 6 = 360$$

3.

(a)

$$\sum_{k=1}^n 3^k \text{ for } n = 1, 2, 3, 4$$

$$3; 3+9=12; 3+9+27=39; 3+9+27+81=120$$

(b)

$$\sum_{k=3}^n k^3 \text{ for } n = 3, 4, 5$$

$$27; 27+64=91; 27+64+125=216$$

(c)

$$\sum_{j=n}^{2n} j \text{ for } n = 1, 2, 5$$

$$1+2=3; 2+3+4=9; 5+6+7+8+9+10=45$$

5.

(a)

$$\prod_{r=1}^n (r - 3) \text{ for } n = 1, 2, 3, 4, 73$$

$$-2; -2 \bullet -1 = 2; -2 \bullet -1 \bullet 0 = 0; -2 \bullet -1 \bullet 0 \bullet 1 = 0; 0$$

(b)

$$\prod_{k=1}^m \frac{k + 1}{k} \text{ for } m = 1, 2, 3$$

$$2; 2 \bullet 3/2 = 3; 2 \bullet 3/2 \bullet 4/3 = 4$$

In general, for $m \in \mathbb{P}$, the product is $m+1$

7.

$$a_n = \frac{n - 1}{n + 1} \text{ for } n \in \mathbb{P}$$

(a) $0; 1/3; 1/2; 3/5; 2/3; 5/7$

(b)

$$a_2 - a_1 = 1/3 - 0 = 1/3$$

$$a_3 - a_2 = 1/2 - 1/3 = 1/6$$

$$a_4 - a_3 = 3/5 - 1/2 = 1/10$$

(c)

$$a_{n+1} - a_n = \frac{(n+1) - 1}{(n+1) + 1} - \frac{n - 1}{n + 1} = \frac{n(n+1) - (n-1)(n+2)}{(n+1)(n+2)} = \frac{2}{(n+1)(n+2)}$$

9. $SEQ(n) = n^2 - n$ for $n \in \mathbb{N}$

(a)

$$SEQ(0) = 0;$$

$$SEQ(1) = 0;$$

$$SEQ(2) = 4 - 2 = 2;$$

$$SEQ(3) = 9 - 3 = 6;$$

$$SEQ(4) = 16 - 4 = 12;$$

$$SEQ(5) = 25 - 5 = 20;$$

$$SEQ(6) = 36 - 6 = 30$$

(b) $SEQ(n+1) = (n+1)^2 - (n+1) = n^2 + 2n + 1 - n - 1 = 2n + SEQ(n)$

(c)

$$SEQ(n+1) = n^2 + n = n(n+1) = (n(n+1)(n-1))/(n-1) = ((n+1)/(n-1)) \bullet SEQ(n)$$

this is valid for $n \geq 2$.