Homework Solutions - Section 2.2

1.

(a) $p \rightarrow (q \wedge r)$

converse: $(q \wedge r) \rightarrow p$

contrapositive: $\neg(q \wedge r) \rightarrow \neg p$

(b) $((x + y) = 1) \rightarrow (x^2 + y^2 \geq 1)$

converse: $(x^2 + y^2 \geq 1) \rightarrow ((x + y) = 1)$

contrapositive: $\neg(x^2 + y^2 \geq 1) \rightarrow \neg((x + y) = 1)$

or if $x^2 + y^2 < 1$, then $(x + y) \neq 1$

(c) $(2 + 2 = 4) \rightarrow (3 + 3 = 8)$

converse: $(3 + 3 = 8) \rightarrow (2 + 2 = 4)$

contrapositive: $\neg(3 + 3 = 8) \rightarrow \neg(2 + 2 = 4)$

or if $(3 + 3 \neq 8)$, then $(2 + 2 \neq 4)$

3. Given: $p \rightarrow q$

(a) converse: $q \rightarrow p$

(b) contrapositive: $\neg q \rightarrow \neg p$

(c) logically equivalent: $\neg q \rightarrow \neg p$; $\neg p \vee q$;

5. Given: $p \rightarrow q$ is 0 (false). This implies p is 1 (true) and q is 0 (false)

(a) $p \wedge q = 0$ (false)

(b) $p \vee q = 1$ (true)

(c) $q \rightarrow p = 1$ (true)

7.

p	q	$\neg(p \wedge q)$	$\neg(p \vee q)$	$\neg p \wedge \neg q$	$\neg p \vee \neg q$
0	0	1	1	1	1
0	1	1	0	0	1
1	0	1	0	0	1
1	1	0	0	0	0

9.

	p	q	r	[(p ∨ q) ∧ r] → (p ∧ ¬q)				
	0	0	0	0	0	**1**	0	1
	0	0	1	0	0	**1**	0	1
	0	1	0	1	0	**1**	0	0
	0	1	1	1	1	**0**	0	0
	1	0	0	1	0	**1**	1	1
	1	0	1	1	1	**1**	1	1
	1	1	0	1	0	**1**	0	0
	1	1	1	1	1	**0**	0	0
step	1	1	1	2	3	**4**	3	2

19.
(a)

	p	q	r	[(p → r) ∧ (q → r)] ↔ [(p ∨ q) → r]					
	0	0	0	1	1	1	**1**	0	1
	0	0	1	1	1	1	**1**	0	1
	0	1	0	1	0	0	**1**	1	0
	0	1	1	1	1	1	**1**	1	1
	1	0	0	0	0	1	**1**	1	0
	1	0	1	1	1	1	**1**	1	1
	1	1	0	0	0	0	**1**	1	0
	1	1	1	1	1	1	**1**	1	1
step	1	1	1	2	3	2	**4**	2	3

(b)

	p	q	r	[(p ∧ q) → r] ↔ [p → (q → r)]				
	0	0	0	0	1	**1**	1	1
	0	0	1	0	1	**1**	1	1
	0	1	0	0	1	**1**	1	0
	0	1	1	0	1	**1**	1	1
	1	0	0	0	1	**1**	1	1
	1	0	1	0	1	**1**	1	1
	1	1	0	1	0	**1**	0	0
	1	1	1	1	1	**1**	1	1
step	1	1	1	2	3	**4**	3	2

(c)

	p	q	(p → q) ↔ [(p ∧ ¬q) → **0**]					
	0	0	1	**1**	0	1	1	0
	0	1	1	**1**	0	0	1	0
	1	0	0	**1**	1	1	0	0
	1	1	1	**1**	0	0	1	0
step	1	1	2	**5**	3	2	4	1