

Homework Solutions - Section 2.5

1.

(a) $(p \vee q) \wedge s \Leftrightarrow (q \vee p) \wedge s$ reason: commutative law 2a

(b) $s \rightarrow \neg(p \vee q) \Leftrightarrow s \rightarrow (\neg p \wedge \neg q)$ reason; DeMorgan's law 8a

(c) $(p \rightarrow q) \wedge (p \vee q) \Leftrightarrow (\neg p \vee q) \wedge (\neg\neg p \vee q)$

reason: implication rule 10a and double negation rule 1

(d) $t \wedge (s \vee p) \Leftrightarrow t \wedge (p \vee s)$ reason: commutative law 2a

3. $(p \rightarrow s) \vee (\neg s \rightarrow t)$

Reason

(a) $\Leftrightarrow (\neg p \vee s) \vee (s \vee t)$

implication rules 10a and 11a

(b) $\Leftrightarrow [(\neg p \vee s) \vee s] \vee t$

associative law 3a

(c) $\Leftrightarrow [\neg p \vee (s \vee s)] \vee t$

associative law 3a

(d) $\Leftrightarrow (\neg p \vee s) \vee t$

idempotent law 5a

(e) $\Leftrightarrow \neg p \vee (s \vee t)$

associative law 3a

(f) $\Leftrightarrow p \rightarrow (s \vee t)$

implication rule 10a

7.

(a) $\neg(p \rightarrow q) \rightarrow ((p \rightarrow q) \rightarrow p)$

(b) $[p \wedge (p \rightarrow (p \rightarrow q))] \rightarrow (p \rightarrow q)$

(c) $p \vee \neg p$

(d) $(p \vee (p \rightarrow q)) \leftrightarrow [(\neg(p \rightarrow q)) \rightarrow p]$

9.

(a) exportation rule 14 with $q \wedge r$ replacing q

(b) distributive law 4a with $r \wedge s$ replacing r

(c) DeMorgan's law 8a

17.

(a) see implication rules 11a and 11b

(b) see DeMorgan's law 8c and implication rule 10b

(c) No, you must have negation.

Note: when $p = 0$ and $q = 0$, $p \rightarrow q$ is true; whereas, any proposition consisting of only p , q , \wedge , and \vee will be false.