

## Homework Solutions - Section 2.6

1.

Given:

$C$

$A \rightarrow C$

Conclusion:

$A$

Attempted Proof

Reason

1.  $A \rightarrow C$

hypothesis

2.  $\neg A \rightarrow \neg C$

INVALID STATEMENT

3.  $C \rightarrow A$

2; contrapositive rule 9

4.  $C$

hypothesis

5.  $A$

3,4; modus ponens rule 30

Hence, the argument is INVALID

5.

(a)

Let,

$B$  = "the TV set is broken"

$S$  = "I will study"

$P$  = "I will pass the course"

Given:

1.  $\neg B \rightarrow \neg S$

2.  $S \rightarrow P$

3.  $\neg P$

Possible Conclusion:

4.  $\neg S$

2,3; modus tollens rule 31

5.  $S \rightarrow B$

1; contrapositive rule 9

6.  $(S \rightarrow B) \wedge (S \rightarrow P)$

5,2; conjunction rule 34

7.  $S \rightarrow (B \wedge P)$

6; implication rule 12b

(b)

Let,

M = "I passed the midterm"

F = "I passed the final"

P = "I passed the course"

Given:

1.  $(M \wedge F) \rightarrow P$

2.  $P \rightarrow F$

3.  $\neg P$

Possible Conclusion:

4.  $\neg(M \wedge F)$       1,3; modus tollens rule 31

5.  $\neg M \vee \neg F$       4; DeMorgan's Law rule 8b

(c)

Let,

M = "I passed the midterm"

F = "I passed the final"

P = "I passed the course"

N = "I will take the next course"

Given:

1.  $(M \vee F) \rightarrow P$

2.  $N \rightarrow P$

3.  $\neg N$

Possible Conclusion:

4.  $(M \rightarrow P) \wedge (F \rightarrow P)$       1; implication rule 12a

5.  $M \rightarrow P$       4; simplification rule 29

6.  $F \rightarrow P$       4; simplification rule 29

9.

(a)

Given:

$$(c \wedge b) \rightarrow r$$

$$\neg b \rightarrow \neg p$$

Conclusion:

$$(\neg r \wedge p) \rightarrow \neg c$$

- |  |                                     |
|--|-------------------------------------|
| 1. $(c \wedge b) \rightarrow r$              | hypothesis                          |
| 2. $\neg b \rightarrow \neg p$               | hypothesis                          |
| 3. $\neg(c \wedge b) \vee r$                 | 1; implication rule 10a             |
| 4. $\neg c \vee \neg b \vee r$               | 3; DeMorgan's law rule 8b           |
| 5. $\neg c \vee r \vee \neg b$               | 4; commutative law rule 2a          |
| 6. $\neg(\neg c \vee r) \rightarrow \neg b$  | 5; implication rule 11a             |
| 7. $\neg(\neg c \vee r) \rightarrow \neg p$  | 6,2; hypothetical syllogism rule 33 |
| 8. $(\neg c \vee r) \vee \neg p$             | 7; implication rule 10a             |
| 9. $\neg c \vee (r \vee \neg p)$             | 8; associative law rule 3a          |
| 10. $c \rightarrow (r \vee \neg p)$          | 9; implication rule 11a             |
| 11. $\neg(r \vee \neg p) \rightarrow \neg c$ | 10; contrapositive rule 9           |
| 12. $(\neg r \wedge p) \rightarrow \neg c$   | 11; DeMorgan's law rule 8a          |

(c)

Given:

$$(j \wedge w) \rightarrow p$$

$$p \rightarrow h$$

$$\neg h$$

Conclusion:

$$\neg j \vee \neg w$$

- |                                 |                            |
|---------------------------------|----------------------------|
| 1. $(j \wedge w) \rightarrow p$ | hypothesis                 |
| 2. $p \rightarrow h$            | hypothesis                 |
| 3. $\neg h$                     | hypothesis                 |
| 4. $\neg p$                     | 2,3; modus tollens rule 31 |
| 5. $\neg(j \wedge w)$           | 1,4; modus tollens rule 31 |
| 6. $\neg j \vee \neg w$         | 5; DeMorgan's law rule 8b  |

13.

(a)

Given:

$$A \rightarrow P$$

$$A \wedge \neg B$$

Conclusion:

$$P \wedge \neg B$$

1. $A \rightarrow P$	hypothesis
2. $A \wedge \neg B$	hypothesis
3. $A$	2; simplification rule 29
4. $P$	1,3; modus ponens rule 30
5. $\neg B$	2; simplification rule 29
6. $P \wedge \neg B$	4,5; conjunction rule 34

(b)

Given:

$$H \wedge \neg R$$

$$(H \wedge N) \rightarrow R$$

Conclusion:

$$\neg N \quad (\text{proof by contradiction})$$

1. $H \wedge \neg R$	hypothesis
2. $(H \wedge N) \rightarrow R$	hypothesis
3. $N$	negation of conclusion
4. $H$	1; simplification rule 29
5. $(H \wedge N)$	4,3; conjunction rule 34
6. $R$	2,5; modus ponens rule 30
7. $\neg R$	1; simplification rule 29
8. $(R \wedge \neg R)$	6,7; conjunction rule 34
9. contradiction	8; rule 7b