Homework Solutions - Section 3.1

1. $S = \{0,1,2,3\}$

(a) $(m,n) \in R_1$ if $m + n = 3$

$R_1 = \{(0,3),(1,2),(2,1),(3,0)\}$

Properties: AR and S

(b) $(m,n) \in R_2$ if $m - n$ is even

$R_2 = \{(0,0),(0,2),(1,1),(1,3),(2,0),(2,2),(3,1),(3,3)\}$

Properties: R, S, and T

(c) $(m,n) \in R_3$ if $m \leq n$

$R_3 = \{(0,0),(0,1),(0,2),(0,3),(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}$

Properties: R, AS, and T

(d) $(m,n) \in R_3$ if $m + n \leq 4$

$R_3 =$
$\{(0,0),(0,1),(0,2),(0,3),(1,0),(1,1),(1,2),(1,3),(2,0),(2,1),(2,2),(3,0),(3,1)\}$

Properties: S

(e) $(m,n) \in R_5$ if $\max\{m, n\} = 3$

$R_5 = \{(0,3),(1,3),(2,3),(3,3),(3,2),(3,1),(3,0)\}$

Properties: S

3. $A = \{0,1,2\}$

(a) $\{(m,n) : m \leq n\} = \{(0,0),(0,1),(0,2),(1,1),(1,2),(2,2)\}$ - Properties: R

(b) $\{(m,n) : m < n\} = \{(0,1),(0,2),(1,2)\}$ - Properties: Neither

(c) $\{(m,n) : m = n\} = \{(0,0),(1,1),(2,2)\}$ - Properties: R, S

(d) $\{(m,n) : mn = 0\} = \{(0,0),(0,1),(0,2),(1,0),(2,0)\}$ - Properties: S

(e) $\{(m,n) : mn=m\} = \{(0,0),(0,1),(0,2),(1,1),(2,1)\}$ - Properties: Neither

(f) $\{(m,n) : m+n \in A\}=\{(0,0),(0,1),(0,2),(1,0),(1,1),(2,0)\}$ - Properties: S

(g) $\{(m,n) : m^2 + n^2 = 2\} = \{(1,1)\}$ - Properties: S

(h) $\{(m,n) : m^2 + n^2 = 3\} = \varnothing$ - Properties: S

(i) $\{(m,n) : m = \max\{n,1\}\} = \{(1,0),(1,1),(2,2)\}$ - Properties: Neither

9. Given a non-empty set S
(a) empty relation = $\varnothing$ = { }

Properties: AR, S, AS, and T

(the last three hold vacuously - the antecedent is false)

(b) universal relation = $S \times S$

Properties: R, S, and T

11. Yes, by the definition of these properties

13. R_1 and R_2 are relations on S

(a)

If R_1 and R_2 are reflexive, then for every $x \in S$, $(x,x) \in R_1$ and $(x,x) \in R_2$. Thus, $(x,x) \in R_1 \cap R_2$. Hence, $R_1 \cap R_2$ is reflexive.

(b)

For every $(x,y) \in R_1 \cap R_2$, $(x,y) \in R_1$ and $(x,y) \in R_2$. If R_1 and R_2 are symmetric, then $(y,x) \in R_1$ and $(y,x) \in R_2$. Thus, $(y,x) \in R_1 \cap R_2$. Hence, $R_1 \cap R_2$ is symmetric.

(c)

For every $(x,y),(y,z) \in R_1 \cap R_2$, $(x,y),(y,z) \in R_1$ and $(x,y),(y,z) \in R_2$.
If R_1 and R_2 are transitive, then $(x,z) \in R_1$ and $(x,z) \in R_2$.
Thus, $(x,z) \in R_1 \cap R_2$. Hence, $R_1 \cap R_2$ is transitive.