Homework Solutions - Section 3.3

3.

$$\boldsymbol{A} = \begin{bmatrix} -1 & 0 & 2 \\ 1 & 3 & -2 \\ 4 & 2 & 3 \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 6 & 8 & 5 \\ 4 & -2 & 7 \\ 3 & 1 & 2 \end{bmatrix} \quad \boldsymbol{C} = \begin{bmatrix} 1 & 3 \\ 2 & -4 \\ 5 & -2 \end{bmatrix}$$

$$\boldsymbol{A}^{T} = \begin{bmatrix} -1 & 1 & 4 \\ 0 & 3 & 2 \\ 2 & -2 & 3 \end{bmatrix} \quad \boldsymbol{C}^{T} = \begin{bmatrix} 1 & 2 & 5 \\ 3 & -4 & 2 \end{bmatrix} \quad \boldsymbol{A}+\boldsymbol{B} = \begin{bmatrix} 5 & 8 & 7 \\ 5 & 1 & 5 \\ 7 & 3 & 5 \end{bmatrix}$$

$$(\boldsymbol{A}+\boldsymbol{B})^{T} = \boldsymbol{A}^{T}+\boldsymbol{B}^{T} = \begin{bmatrix} 5 & 5 & 7 \\ 8 & 1 & 3 \\ 7 & 5 & 5 \end{bmatrix} \quad \boldsymbol{B}+\boldsymbol{B}^{T} = \begin{bmatrix} 12 & 12 & 8 \\ 12 & -4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$$

$\boldsymbol{A}+\boldsymbol{C}$ *does not exist*

$\boldsymbol{C}+\boldsymbol{C}^{T}$ *does not exist*

5.

$\mathbf{A} = [a_{ij}]$ with $a_{ij} = (-1)^{i+j}$ and $\mathbf{B} = [b_{ij}]$ with $b_{ij} = i + j$ in $\mathfrak{M}_{4,3}$

$$\boldsymbol{A} = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \\ 5 & 6 & 7 \end{bmatrix}$$

$$\boldsymbol{A}^T = \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \qquad \boldsymbol{A}+\boldsymbol{B} = \begin{bmatrix} 3 & 2 & 5 \\ 2 & 5 & 4 \\ 5 & 4 & 7 \\ 4 & 7 & 6 \end{bmatrix} \qquad \boldsymbol{A}^T+\boldsymbol{B} \textit{ does not exist}$$

$$\boldsymbol{A}^T+\boldsymbol{B}^T = (\boldsymbol{A}+\boldsymbol{B})^T = \begin{bmatrix} 3 & 2 & 5 & 4 \\ 2 & 5 & 4 & 7 \\ 5 & 4 & 7 & 6 \end{bmatrix} \qquad \boldsymbol{A}+\boldsymbol{A} = \begin{bmatrix} 2 & -2 & 2 \\ -2 & 2 & -2 \\ 2 & -2 & 2 \\ -2 & 2 & -2 \end{bmatrix}$$

11.

(a)

$$\boldsymbol{A_n} = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \qquad \boldsymbol{B_n} = \begin{bmatrix} 1 & (-1)^n \\ -1 & 1 \end{bmatrix}$$

$$\boldsymbol{A_n^T} = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} \qquad \boldsymbol{B_n^T} = \begin{bmatrix} 1 & -1 \\ (-1)^n & 1 \end{bmatrix}$$

(b) $\{n \in \mathbb{N} : \mathbf{A_n}^{\mathbf{T}} = \mathbf{A_n}\} = \{0\}$

(c) $\{n \in \mathbb{N} : \mathbf{B_n}^{\mathbf{T}} = \mathbf{B_n}\} = \{n \in \mathbb{N} : n \text{ is odd}\}$

(d) $\{n \in \mathbb{N} : \mathbf{B_n} = \mathbf{B_0}\} = \{n \in \mathbb{N} : n \text{ is even}\}$

15.

$$(\boldsymbol{a}) \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad (\boldsymbol{b}) \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (\boldsymbol{c}) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$