Homework Solutions - Section 3.5

1.

(a) 20/3	q = 6	r = 2
(b) 20/4	q = 5	r = 0
(c) -20/3	q = -7	r = 1
(d) -20/4	q = -5	r = 0
(e) 371246/65	q = 5711	r = 31
(f) -371246/65	q = -5712	r = 34

3.
(a) -4,0,4,8,...
(b) -3,1,5,9,...
(c) -2,2,6,10,...
(d) -1,3,7,11,...
(e) -4,0,4,8,12,...

5. We simply calculate the remainder of each number when divided by 4
(a) 17/4 has a remainder of 1
(b) 7/4 has a remainder of 3
(c) -7/4 has a remainder 1
(d) 2/4 has a remainder of 2
(e) -88/4 has a remainder of 0

15. N = abcd

(a) You can write n as:

$$n = 1000a + 100b + 10\,c + d$$
$$= 999a + 99b + 9c + (a + b + c + d)$$

Now 999a + 99b + 9c is a multiple of 9.

Hence, n will be a multiple of 9 iff (a + b + c + d) is a multiple of 9.

(b) Yes.

For any n ∈ ℙ that has m digits, we can write:

$$n = \sum_{k=0}^{m} a_k 10^k = \sum_{k=0}^{m} a_k(10^k - 1) + \sum_{k=0}^{m} a_k$$

Now, $\sum_{k=0}^{m} a_k(10^k - 1)$ is a multiple of 9.

Hence, n is a multiple of 9 iff $\sum_{k=0}^{m} a_k$ is a multiple of 9.