Homework Solutions - Section 4.4

1. $s_0 = 1$, $s_{n+1} = 2/s_n$ for $n \in \mathbb{N}$
(a) $s_0 = 1$, $s_1 = 2/s_0 = 2/1 = 2$, $s_2 = 2/s_1 = 2/2 = 1$, $s_3 = 2/s_2 = 2/1 = 2$, ...
(b) $\{1, 2\}$

3. (1, 3, 9, 27, 81, ...)
(a) $\text{SEQ}(n) = 3^n$ for $n \in \mathbb{N}$
(b)
(B) $\text{SEQ}(0) = 1$
(R) $\text{SEQ}(n) = 3 \cdot \text{SEQ}(n-1)$ for $n \geq 1$

7. $\Sigma = \{a, b, c\}$
(a)
$s_0 = 1$; namely $\{\lambda\}$
$s_1 = 3$; namely {a, b, c}
$s_2 = 8$; namely {ab, ac, ba, bb, bc, ca, cb, cc}
(b)
Part (a) forms the basis.
To form a word of length n that does not end in ‘aa’, you can either start with any word of length n-1 that does not end in ‘aa’ and append either a ‘b’ or a ‘c’ (there are $2 \cdot s_{n-1}$ such words of length n), or , to create a word of length n that does not end in ‘aa’ but does end in ‘a’, you can start with any word of length n-2 that does not end in ‘aa’ and append either ‘ba’ or ‘ca’ (there are $2 \cdot s_{n-2}$ such words of length n).
Hence, $s_n = 2 \cdot s_{n-1} + 2 \cdot s_{n-2}$ for $n \geq 2$
(c) $s_3 = 2s_2 + 2s_1 = 2(8) + 2(3) = 22$; $s_4 = 2s_3 + 2s_2 = 2(22) + 2(8) = 60$

9. $\Sigma = \{a, b\}$

(a)

$t_0 = 1$; namely $\{\lambda\}$

$t_1 = 1$; namely {b}

$t_2 = 2$; namely {aa, bb}

$t_3 = 4$; namely {aab, aba, baa, bbb}

(b)

To form a word of length n with an even number of a's, either start with any word of length n-1 that has an even number of a's and append a 'b' (there are t_{n-1} such words), or start with a word of length n-1 that has an odd number of a's and append a 'a' (there are $2^{n-1} - t_{n-1}$ such words).

Hence, $t_n = t_{n-1} + (2^{n-1} - t_{n-1}) = 2^{n-1}$ for $n \geq 1$

(c) No

17.

(a)

(B) $A(1) = 1$

(R) $A(n) = n \cdot A(n-1)$

(b)

$A(6) = 6 \cdot A(5) = 6 \cdot 5 \cdot A(4) = 6 \cdot 5 \cdot 4 \cdot A(3) = 6 \cdot 5 \cdot 4 \cdot 3 \cdot A(2) = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot A(1)$

$= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6! = 720$

(c) Yes, $A(n) = n!$