Homework Solutions - Section 5.1

1.

(a) $\binom{8}{3} = \dfrac{8!}{5!\cdot 3!} = \dfrac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1} = 56$

(b) $\binom{8}{0} = \dfrac{8!}{8!\cdot 0!} = 1$

(c) $\binom{8}{5} = \dfrac{8!}{3!\cdot 5!} = 56$

(d) $\binom{52}{50} = \dfrac{52!}{2!\cdot 50!} = \dfrac{52\cdot 51}{2\cdot 1} = 1326$

(e) $\binom{52}{52} = \dfrac{52!}{0!\cdot 52!} = 1$

(f) $\binom{52}{1} = \dfrac{52!}{51!\cdot 1!} = \dfrac{52}{1} = 52$

3.

(a) $P(10,1) = \dfrac{10!}{9!} = 10$

(b) $P(10,0) = \dfrac{10!}{10!} = 1$

(c) $P(10,-2) + P(10,17) = 0 + 0 = 0$

(d) $P(100,40)/P(99,39) = (\dfrac{100!}{60!})/(\dfrac{99!}{60!}) = \dfrac{100!}{99!} = 100$

(e) $P(1000,350)/P(999,350) = (\dfrac{1000!}{650!})/(\dfrac{999!}{649!}) = \dfrac{1000}{650}$

(f) $P(100,2) = \dfrac{10!0}{98!} = 100\cdot 99 = 9900$

7.

(a) $\binom{9}{4} = \dfrac{9!}{5!\cdot 4!} = \dfrac{9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2\cdot 1} = 126$

(b) $\binom{7}{4}\binom{2}{0} + \binom{7}{3}\binom{2}{1} = \dfrac{7!}{3!\cdot 4!} + (\dfrac{7!}{4!\cdot 3!})(2) = \dfrac{7\cdot 6\cdot 5}{3\cdot 2\cdot 1}(3) = 105$

9.

$S = \{a,b,c,d\}$ $T = \{1,2,3,4,5,6,7\}$

(a) 0

For a one-to-one function, each element of T must map onto a different element of S. However, there are not enough elements in S.

(b)

For a one-to-one function, each element of S must map onto a different element of T. The number of such permutations is:

$$P(7,4) = \frac{7!}{3!} = 7 \cdot 6 \cdot 5 \cdot 4 = 840$$

(c)

Each element of S can map onto each element of T. The number of such mappings is:

$$7 \cdot 7 \cdot 7 \cdot 7 = (7)^4 = 2401$$

11.

(a) This is the same as the number of ways of drawing ten cards so that the first one is not a repetition.

(We get an ordered 10-tuple XYYYYYYYYY)

The number of ways to do this is:

$$52 \cdot 51 \cdot 51 \cdot 51 \cdot 51 \cdot 51 \cdot 51 \cdot 51 \cdot 51 \cdot 51 = 52 \cdot (51)^9$$

(b) This is all the remaining ways of drawing ten cards:

$$52^{10} - (52 \cdot 51^9)$$

15.

(a) Represent the hand as and ordered pair (x,y) where x is the four of a kind and y is the fifth card. There are $\binom{13}{1}\binom{4}{4} = 13$ ways to choose x and $\binom{48}{1} = 48$ ways to choose y. Therefore, there are 13 · 48 = 624 hands of four of a kind.

(b) In each of the four suits , there are $\binom{13}{5} = \frac{13!}{8!\cdot 5!} = \frac{13\cdot 12\cdot 11\cdot 10\cdot 9}{5\cdot 4\cdot 3\cdot 2\cdot 1} = 1287$ flushes of which 10 are straight flushes. Thus, there are a total of 4 (1287 - 10) = 5108 flushes that are not straight flushes.

(c) Represent the hand as and ordered pair (x,{y,z}) where x is the three of a kind and y and z are the fourth and fifth cards. There are $\binom{13}{1}\binom{4}{3} = 13\cdot 4 = 52$ ways to choose x and $\binom{12}{2}\binom{4}{1}\binom{4}{1} = \frac{12!}{10!\cdot 2!}\cdot 4\cdot 4 = 66\cdot 4\cdot 4 = 1056$ ways to choose y and z. Hence, there are 52 · 1056 = 54,912 three of a kind hands.

(d) Represent the hand as and ordered pair (w,{x,y,z}) where w is the pair and x,y,z are the third, fourth and fifth cards. There are $\binom{13}{1}\binom{4}{2} = 13\cdot 6 = 78$ ways to choose w and $\binom{12}{3}\binom{4}{1}\binom{4}{1}\binom{4}{1} = \frac{12!}{9!\cdot 3!}\cdot 4\cdot 4 = 220\cdot 4\cdot 4\cdot 4 = 14{,}080$ ways to choose x,y and z. Hence, there are 78 · 14,080 = 1,098,240 hands of one pair.