Homework Solutions - Section 5.2

1. $\{1,2,3,...,25\}$
(a) There are $\lfloor 25/3 \rfloor = 8$ numbers in the set divisible by 3. Thus, the probability is $8/25 = 0.32$
(b) There are $\lfloor 25/5 \rfloor = 5$ numbers in the set divisible by 5. Thus, the probability is $5/25 = 0.2$
(c) The primes are $\{2,3,5,7,11,13,17,19,23\}$. Thus, the probability is $9/25 = 0.36$

3. $\Sigma = \{a,b,c,d,e\}$
(a) $|\Sigma^4| = 5^4 = 625$
The number of words with distinct letters is: $P(5,4) = \frac{5!}{1!} = 5 \cdot 4 \cdot 3 \cdot 2 = 120$
Thus, the probability is $120/625 = 0.192$
(b) The number of words with no vowels is $3^4 = 81$.
Thus, the probability is $81/625 = 0.1296$
(c) The number of words that begin with a vowel is $2 \cdot 5 \cdot 5 \cdot 5 = 250$
Thus, the probability is $250/625 = 0.4$

5. Urn with 3 red balls and 4 black balls.
Note: the total number of ways of choosing 3 balls from 7 is

$$\binom{7}{3} = \frac{7!}{4!\cdot 3!} = \frac{7\cdot 6\cdot 5}{3\cdot 2\cdot 1} = 35$$

(a) The number of ways to choose 3 red and 0 black is: $\binom{3}{3}\binom{4}{0} = 1\cdot 1 = 1$

Thus, the probability is 1/35

(b) The number of ways to choose 0 red and 3 black is: $\binom{3}{0}\binom{4}{3} = 1\cdot 4 = 4$

Thus, the probability is 4/35

(c) The number of ways to choose 1 red and 2 black is: $\binom{3}{1}\binom{4}{2} = 3\cdot 6 = 18$

Thus, the probability is 18/35

(d) The number of ways to choose 2 red and 1 black is: $\binom{3}{2}\binom{4}{1} = 3\cdot 4 = 12$

Thus, the probability is 12/35

(e) 1/35 + 4/35 + 18/35 + 12/35 = 1 (as expected)

7. $P(A) = 0.5$; $P(B) = 0.8$; $P(A \cap B) = 0.4$

(a) $P(B^c) = 1 - P(B) = 1 - 0.8 = 0.2$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.8 - 0.4 = 0.9$

(c) $P(A^c \cup B^c) = P(A \cap B)^c = 1 - 0.4 = 0.6$ (using DeMorgan's law)

9. We will use the results from Section 5.1 example 10 and problem 15

Note: Total number of possible hands is $\binom{52}{5} = \frac{52!}{47!\cdot 5!} = 2{,}598{,}960$

(a) 624/2,589,960 ≈ 0.000240

(b) 54,9124/2,589,960 ≈ 0.0211

(c) 10,200/2,589,960 ≈ 0.0392

(d) 123,552/2,589,960 ≈ 0.0475

(e) 1,098,240/2,598,960 ≈ 0.423

15. There are 2^6 possible outcomes

(a) There are $\binom{6}{0} = 1$ way of getting no heads.

The probability of no heads is 1/64

(b) There are $\binom{6}{1} = 6$ ways to getting 1 head.

The probability of 1 head is 6/64 = 3/32

(c) There are $\binom{6}{2} = 15$ ways to getting 2 heads.

The probability of 2 heads is 15/64

(d) There are $\binom{6}{3} = 20$ ways to getting 3 heads.

The probability of 3 heads is 20/64 = 5/16

(e) P(> 3 heads) = 1 - P(≤ 3 heads) = 1 - (1+6+15+20)/64 = 22/64

19. S = {1,2,3,4,5,6,7,8}

Note: there are $\binom{8}{4} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = 70$ ways of choosing 4 numbers from S

(a) There are 4 even and 4 odd numbers. Therefore, there are

$\binom{4}{2}\binom{4}{2} = 6 \cdot 6 = 36$ ways of choosing exactly 2 even numbers.

Thus, the probability is 36/70 = 18/35

(b) There is $\binom{4}{0}\binom{4}{4} = 1 \cdot 1 = 1$way of choosing 0 even numbers.

Thus, the probability is 1/70

(c) There are $\binom{4}{1}\binom{4}{3} = 4 \cdot 4 = 16$ ways of choosing 1 even number.

Thus, the probability is 16/70 = 4/35

(d) There are $\binom{4}{3}\binom{4}{1} = 4 \cdot 4 = 16$ ways of choosing 3 even numbers.

Thus, the probability is 16/70 = 4/35

(e) There is $\binom{4}{4}\binom{4}{0} = 1 \cdot 1 = 1$way of choosing 0 even numbers.

Thus, the probability is 1/70