Homework Solutions - Section 5.3

1.
$|S \cup J| = |S| + |J| - |S \cap J| = 85 + |J| - 60 = 150$
$|J| = 125$

3. S = {1,2,3,...,1000}
(a) $|D_7| = \lfloor 1000/7 \rfloor = 142$
Thus, the probability is 142/1000 = 0.142
(b) $|D_{11}| = \lfloor 1000/11 \rfloor = 90$
Thus, the probability is 90/1000 = 0.09
(c) $|D_7 \cap D_{11}| = \lfloor 1000/77 \rfloor = 12$
$|D_7 \cup D_{11}| = |D_7| + |D_{11}| - |D_7 \cap D_{11}| = 142 + 90 - 12 = 220$
$|(D_7 \cup D_{11})^c| = 1000 - |D_7 \cup D_{11}| = 1000 - 220 = 780$
Thus, the probability is 780/1000 = 0.78
(d) $|D_7 \oplus D_{11}| = |D_7 \cup D_{11}| - |D_7 \cap D_{11}| = 220 - 12 - 208$
Thus, the probability is 208/1000 = 0.208

7.
There are $\binom{n+k-1}{k-1} = \binom{n+k-1}{n}$ ways to place n identical objects into k distinguishable boxes.
(a) with n = 12 and k = 4

$$\binom{n+k-1}{k-1} = \binom{12+4-1}{4-1} = \binom{15}{3} = \frac{15 \cdot 14 \cdot 13}{3 \cdot 2 \cdot 1} = 455$$

(b) Put 2 letters into each box and distribute the remaining 4 letters;
Thus, with n = 4 and k = 4, we get: $\binom{4+4-1}{4-1} = \binom{7}{3} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35$

9.
(a) $(x+2y)^4 = x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$
(b) $(x-y)^6 = x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6$
(c) $(3x+1)^4 = 81x^4 + 108x^3 + 54x^2 + 12x + 1$
(d) $(x+2)^5 = x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$

15.

(a) With only 14 objects, only one box can have at least 8 objects. Hence, pick the box that has at least 8 objects (there are 3 way to do this) and distribute the remaining 6 objects over the 3 boxes.

Thus we get: $3\cdot\binom{6+3-1}{3-1} = 3\cdot\binom{8}{2} = \frac{3\cdot8\cdot7}{2\cdot1} = 84$

(b) To solve, we subtract the result of part (a) from the total number of ways to distribute 14 objects over 3 boxes.

Thus we get: $\binom{14+3-1}{3-1} - 84 = \binom{16}{2} - 84 = 120 - 84 = 36$

(c) The problem can be viewed as having to distribute 20 objects over 3 boxes (digit positions). Now, since $1+9+9 < 20$, each digit (box) must be at least 2. Therefore, this leaves 14 objects to distribute over the three digit positions. Thus, the result is as in part (b) which is 36.

17.

(a) We can view the problem as distributing 9 objects over 4 boxes with at least 1 object in the leftmost box.

Thus, we get: $\binom{8+4-1}{4-1} = \binom{11}{3} = \frac{11\cdot10\cdot9}{3\cdot2\cdot1} = 165$

(b) We now start with 1 object in each box and distribute the remaining 5 over the 4 boxes. Thus, we get: $\binom{5+4-1}{4-1} = \binom{8}{3} = \frac{8\cdot7\cdot6}{3\cdot2\cdot1} = 56$