Homework Solutions - Section 5.4

1.

(a) The committees are distinguishable by their size. Therefore, we can view the committees as an ordered partition (A, B, C, D) of committees. (Note: D represents those people not on any committee)

Thus, we get: $\binom{15}{3}\binom{12}{4}\binom{8}{5}\binom{3}{3} = \frac{15!}{3!4!5!3!}$

(b) Since all 15 people are eligible for each committee, we get:

$$\binom{15}{3}\binom{15}{4}\binom{15}{5}$$

3.

(a) This teams are distinguishable by their size. Thus we count ordered partitions and get: $\binom{13}{5}\binom{8}{3}\binom{5}{2}\binom{3}{3} = \frac{13!}{5!3!2!3!} = 720{,}720$

(b) Two of the teams are not distinguishable.

Thus, we get: $\binom{13}{4}\binom{9}{3}\binom{6}{3}\binom{3}{3}\cdot\frac{1}{2!} = \frac{13!}{4!3!3!3!}\cdot\frac{1}{2!} = 600{,}600$

(c) Now all three teams are indistinguishable.

Thus, we get: $\binom{13}{3}\binom{10}{3}\binom{9}{3}\binom{4}{4}\cdot\frac{1}{3!} = \frac{13!}{4!3!3!3!}\cdot\frac{1}{3!} = 200{,}200$

5.

(a) $3^{10} = 59{,}049$

(b) There are $\binom{10}{5}\binom{5}{5} = \frac{10\cdot9\cdot8\cdot7\cdot6}{5\cdot4\cdot3\cdot2\cdot1} = 252$ such sequences in S

(c) There are $\binom{10}{3}\binom{7}{7} = \frac{10\cdot9\cdot8}{3\cdot2\cdot1} = 120$ such sequences in S

(d) There are $\binom{10}{3}\cdot2^7 = 120\cdot128 = 15{,}360$ such sequences in S

(e) There are $\binom{10}{3}\binom{7}{4}\binom{3}{3} = \frac{10!}{3!4!3!} = 120\cdot35 = 4200$ such sequences in S

(f)

Let Z = the set of sequences with no 0's

Let O = the set of sequences with no 1's

Let T = the set of sequences with no 2's

$|Z \cup O \cup T| = |Z|+|O|+|T| - |Z \cap O| - |Z \cap T| - |O \cap T|+|Z \cap O \cap T|$

$= 3 \cdot 2^{10} - 3 \cdot 1 + 0 = 3{,}069$

$|(Z \cup O \cup T)^c| = 3^{10} - |Z \cup O \cup T| = 59{,}049 - 3{,}069 = 55{,}980$

9.

There are $\binom{2n}{n}\binom{n}{n} = \frac{(2n)!}{n!}$ ordered partitions and $\binom{2n}{n}\binom{n}{n}\cdot\frac{1}{2!} = \frac{1}{2}\cdot\frac{(2n)!}{n!}$ unordered partitions

11. The team with Ann and Bob is distinguishable and the other three teams are indistinguishable.

(a) Choose a third contestant for the Ann and Bob team and then partition the remaining 9 contestants.

Thus, we get $\binom{10}{1}\binom{9}{3}\binom{6}{3}\binom{3}{3}\cdot\frac{1}{3!} = \frac{10!}{1!3!3!3!3!} = 2800$ ways

(b) The total number of ways to partition 12 contestants into 4 indistinguishable teams is $\binom{12}{3}\binom{9}{3}\binom{6}{3}\binom{3}{3}\cdot\frac{1}{4!} = \frac{12!}{3!3!3!3!4!} = 15{,}400$

Thus, the probability that Ann and Bob are on the same team is $2800/15{,}400 = 2/11 \approx 0.18$