Homework Solutions - Section 9.1

1. We throw a pair of dice (1 red and 1 black)
(a) $P(B_o) = P(R_o) = P(E) = ½$
$P(B_o \cap R_o) = P(B_o \cap E) = P(E \cap R_o) = ¼$
$P(B_o \cap R_o) = P(B_o)P(R_0)$
$P(B_o \cap E) = P(B_o)P(E)$
$P(E \cap R_o) = P(E)P(R_0)$
which shows that they are pairwise independent
$P(B_o \mid E \cap R_o) = 1 \neq P(B_o)$

3.
$P(S) = 15/36$; $P(L) = 15/36$; $P(E) = 6/36$; $P(G) = 15/36$
$P(S \cap L) = 6/36 \neq P(S)P(L)$
$P(S \cap E) = 3/36 \neq P(S)P(E)$
$P(S \cap G) = 6/36 \neq P(S)P(G)$
$P(L \cap E) = 0 \neq P(L)P(E)$
$P(L \cap G) = 0 \neq P(L)P(G)$
$P(E \cap G) = 0 \neq P(E)P(G)$
Thus, none of the events are pairwise independent

7. $P(A) = 0.3$; $P(B) = 0.4$; $P(C) = 0.8$; $P(A \cap B) = 0.1$
(a) $P(A \mid B) = P(A \cap B)/P(B) = 0.1/0.4 = ¼ = 0.25$
(b) $P(A^c) = 1 - P(A) = 1 - 0.3 = 0.7$
(c) $P(A \mid B) \neq P(A)$
Thus, A and B are not independent
(d) $P(A^c \cap B) = P(B) - P(A \cap B) = 0.4 - 0.1 = 0.3 \neq P(A^c)P(B)$
Thus, A^c and B are not independent

9.

The numbers of ways to choose 2 marbles from 11 is $\binom{11}{2} = \frac{11 \cdot 10}{2 \cdot 1} = 55$

(a) There are $\binom{3}{2}\binom{8}{0} = 3$ ways to choose 2 red balls

The probability is 3/55

(b) There are $\binom{3}{0}\binom{8}{2} = \frac{8 \cdot 7}{2 \cdot 1} = 28$ ways to choose 2 black balls

The probability is 28/55

(c) There are $\binom{3}{1}\binom{8}{1} = 3 \cdot 8 = 24$ ways to choose 2 black balls

The probability is 24/55

13.

(a) $P(B) = P(B|U_1)P(U_1) + P(B|U_2)P(U_2) + P(B|U_2)P(U_2)$

$= ⅔ \cdot ⅓ + 2/5 \cdot ⅓ + ½ \cdot ⅓ = 47/90$

(b)

$P(U_1|B) = P(U_1 \cap B)/P(B) = P(B|U_1)P(U_1)/P(B) = ⅔ \cdot ⅓ /(47/90) = 20/47$

$P(U_2|B) = P(U_2 \cap B)/P(B) = P(B|U_2)P(U_2)/P(B) = 2/5 \cdot ⅓ /(47/90) = 12/47$

$P(U_3|B) = P(U_3 \cap B)/P(B) = P(B|U_3)P(U_3)/P(B) = ½ \cdot ⅓ /(47/90) = 15/47$

(c)

$P(B \cap U_1) = P(B|U_1)P(U_1) = ⅔ \cdot ⅓ = 2/9$

17.

(a) $P(C) = P(C|E)P(E) + P(C|F)P(F) + P(C|G)P(G)$

$= (0.04)(0.25) + (0.06)(0.35) + (0.03)(0.4) = 0.043$

(b)

$P(E|C) = P(E \cap C)/P(C) = (0.04)(0.25)/0.043 = 10/43 \approx 0.23$

$P(F|C) = P(F \cap C)/P(C) = (0.06)(0.35)/0.043 = 21/43 \approx 0.49$

$P(G|C) = P(G \cap C)/P©) = (0.03)(0.4)/0.043 = 12/43 \approx 0.28$

(c) 1

19. $P(N^c \cap D) = 0.004$; $P(N \cap D) = 0.0001$; $P(N^c) = 0.044$

(a)

$P(D) = P(N^c \cap D) + P(N \cap D) = 0.004 + 0.0001 = 0.0041$

$P(N^c \mid D) = P(N^c \cap D)/P(D) = 0.004/0.0041 \approx 0.9756$

(b)

$P(N \cap D^c) = P(N) - P(N \cap D) = 1 - P(N^c) - P(N \cap D) = 0.9559$

$P((N^c \cap D) \cup (N \cap D^c)) = P(N^c \cap D) + P(N \cap D^c) = 0.004+0.9559 \approx 0.96$

(c)

$P(D \mid N^c) = P(D \cap N^c)/P(N^c) = 0.004/0.044 \approx 0.091$