Number Systems

- The total number of allowable symbols in a number system is called the **radix** or **base** of the system.

  - Decimal Numbers:
    radix = 10 (symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

  - Binary Numbers:
    radix = 2 (symbols: 0, 1)

  - Hexadecimal Numbers:
    radix = 16 (symbols: 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F)

- **Positional Notation:**
  The **radix** point separates the *integer* part of the number from the *fractional* part of the number. Each position represents a power of the radix.

  - Example: decimal number

  1 2 5 6 . 9 3 2

  $= (1 \times 10^3) + (2 \times 10^2) + (5 \times 10^1) + (6 \times 10^0)$

  $+ (9 \times 10^{-1}) + (3 \times 10^{-2}) + (2 \times 10^{-3})$

  - Example: binary number

  $(1\ 1\ 0\ 1\ 0\ .\ 1\ 1\ 0\ 1)_2$

  $= (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$

  $+ (1 \times 2^{-1}) + (1 \times 2^{-2}) + (0 \times 2^{-3}) + (1 \times 2^{-4})$

  = 26.8125 = 26 13/16

Binary Numbers

- Assume 1-bit numbers:

  Only two different numbers can be represented:

  0
  1

- Assume 2-bit numbers:

  Four different numbers can be represented:

  00 - 0
  01 - 1
  10 - 2
  11 - 3

- Assume 3-bit numbers:

  Eight different numbers can be represented:

  000 - 0
  001 - 1
  010 - 2
  011 - 3
  100 - 4
  101 - 5
  110 - 6
  111 - 7

Hexadecimal Numbers

- Assume 4-bit binary numbers:

  Sixteen different numbers can be represented:

Binary	Decimal	Hexadecimal
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F

- Sample Hexadecimal Number:

  $(A\ 1\ F\ .\ 1\ C)_{16}$

  $= (A \times 16^2) + (1 \times 16^1) + (F \times 16^0) + (1 \times 16^{-1}) + (C \times 16^{-2})$

  $= (10 \times 16^2) + (1 \times 16^1) + (15 \times 16^0)$

  $\quad + (1 \times 16^{-1}) + (12 \times 16^{-2})$

  $= 2591\ 28/256$

Binary <--> Hexadecimal Conversions

- Example:

  $(01000010101001010110.11110001)_2 = (?)_{16}$

  0100 0010 1010 0101 0110 . 1111 0001

  $= (4\ 2\ A\ 5\ 6\ .\ F\ 1)_{16}$

- Example:

  $(AF5.2C)_{16} = (?)_2$

  A F 5 . 2 C

  $= (1010\ 1111\ 0101\ .\ 0010\ 1100)_2$

- Example:

  $(11010.1101)_2 = (?)_{16}$

  0001 1010 . 1101

  $= (1\ A\ .\ D)_{16}$

Radix Divide Techniques

(Decimal-to-Base x Integer Conversions)

- Divide the given integer successively by the required radix, noting the remainder at each step.
- The quotient at each step becomes the new dividend for the subsequent division.
- Stop the division process when the quotient becomes zero.
- Collect the remainders from each step (last to first) and place them left to right to form the required number.
- **Decimal-to-Binary Conversions:**

  $(245)_{10} = (?)_2$

- **Decimal-to-Hexadecimal Conversions:**

  $(245)_{10} = (?)_{16}$

Arithmetic

- Binary Addition:

```
    0        0        1         1
 +  0        1        0         1
------   ------   ------   --------
    0        1        1        10
```

- Example:

```
101
001
------
```

- Example

```
10110
00111
----------
```

- Hexadecimal Addition:

```
    2        9        7        E          F          F
 +  5        1        5        1          1          2
------   ------   ------   ------   --------   --------
    7        A        C        F         10         11
```

- Example:

```
1 5 F C
2 4 5 D
-----------
```