

Number Systems

- The total number of allowable symbols in a number system is called the **radix** or **base** of the system.

- Decimal Numbers:

radix = 10 (symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

- Binary Numbers:

radix = 2 (symbols: 0, 1)

- Hexadecimal Numbers:

radix = 16 (symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)

- **Positional Notation:**

The **radix** point separates the *integer* part of the number from the *fractional* part of the number. Each position represents a power of the radix.

- **Example: decimal number**

1 2 5 6 . 9 3 2

$$\begin{aligned} &= (1 \times 10^3) + (2 \times 10^2) + (5 \times 10^1) + (6 \times 10^0) \\ &\quad + (9 \times 10^{-1}) + (3 \times 10^{-2}) + (2 \times 10^{-3}) \end{aligned}$$

- **Example: binary number**

(1 1 0 1 0 . 1 1 0 1)₂

$$\begin{aligned} &= (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) \\ &\quad + (1 \times 2^{-1}) + (1 \times 2^{-2}) + (0 \times 2^{-3}) + (1 \times 2^{-4}) \\ &= 26.8125 = 26 \frac{13}{16} \end{aligned}$$

Binary Numbers

- Assume 1-bit numbers:

Only two different numbers can be represented:

0
1

- Assume 2-bit numbers:

Four different numbers can be represented:

00 - 0
01 - 1
10 - 2
11 - 3

- Assume 3-bit numbers:

Eight different numbers can be represented:

000 - 0
001 - 1
010 - 2
011 - 3
100 - 4
101 - 5
110 - 6
111 - 7

Hexadecimal Numbers

- Assume 4-bit binary numbers:

Sixteen different numbers can be represented:

<u>Binary</u>	<u>Decimal</u>	<u>Hexadecimal</u>
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F

- Sample Hexadecimal Number:

$$(A\ 1\ F\ .\ 1\ C)_{16}$$

$$= (A \times 16^2) + (1 \times 16^1) + (F \times 16^0) + (1 \times 16^{-1}) + (C \times 16^{-2})$$

$$= (10 \times 16^2) + (1 \times 16^1) + (15 \times 16^0)$$

$$+ (1 \times 16^{-1}) + (12 \times 16^{-2})$$

$$= 2591\ 28/256$$

Binary < --> Hexadecimal Conversions

- Example:

$$(01000010101001010110.11110001)_2 = (?)_{16}$$

0100 0010 1010 0101 0110 . 1111 0001

$$= (4\ 2\ A\ 5\ 6\ .\ F\ 1)_{16}$$

- Example:

$$(AF5.2C)_{16} = (?)_2$$

A F 5 . 2 C

$$= (1010\ 1111\ 0101\ .\ 0010\ 1100)_2$$

- Example:

$$(11010.1101)_2 = (?)_{16}$$

0001 1010 . 1101

$$= (1\ A\ .\ D)_{16}$$

Radix Divide Techniques

(Decimal-to-Base x Integer Conversions)

- Divide the given integer successively by the required radix, noting the remainder at each step.
- The quotient at each step becomes the new dividend for the subsequent division.
- Stop the division process when the quotient becomes zero.
- Collect the remainders from each step (last to first) and place them left to right to form the required number.
- **Decimal-to-Binary Conversions:**

$$(245)_{10} = (?)_2$$

- **Decimal-to-Hexadecimal Conversions:**

$$(245)_{10} = (?)_{16}$$

Arithmetic

- Binary Addition:

$$\begin{array}{r} 0 \quad 0 \quad 1 \quad 1 \\ + 0 \quad 1 \quad 0 \quad 1 \\ \hline 0 \quad 1 \quad 1 \quad 10 \end{array}$$

- Example:

$$\begin{array}{r} 101 \\ 001 \\ \hline \end{array}$$

- Example

$$\begin{array}{r} 10110 \\ 00111 \\ \hline \end{array}$$

- Hexadecimal Addition:

$$\begin{array}{r} 2 \quad 9 \quad 7 \quad E \quad F \quad F \\ + 5 \quad 1 \quad 5 \quad 1 \quad 1 \quad 2 \\ \hline 7 \quad A \quad C \quad F \quad 10 \quad 11 \end{array}$$

- Example:

$$\begin{array}{r} 15FC \\ 245D \\ \hline \end{array}$$