Algorithms

Assignment: Analysis of Algorithms

Name: ............................................................

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Grade

Good Luck!
1. Rearrange the following 20 functions in a decreasing order of their growth:

\[
\begin{align*}
\log_2(n) & \quad 3^n & \quad n^2 & \quad \sqrt{n} & \quad \frac{n}{\log_2(n)} \\
 n^3 & \quad n! & \quad n & \quad 1000^n & \quad n(\log_2(n))^2 \\
 n^{1000} & \quad n^n & \quad n^{1/1000} & \quad 2^n & \quad (\log_3(n))^3 \\
 n^{1/3} & \quad n \log_2(n) & \quad 1 & \quad (\log_2(n))^2 & \quad \log_2 \log_2(n)
\end{align*}
\]
2. Let \( P \) be a problem. The worst-case time complexity of \( P \) is \( O(n^2) \). The worst-case time complexity of \( P \) is also \( \Omega(n \log n) \). Let \( A \) be an algorithm that solves \( P \). Which subset of the following statements are consistent with this information about the complexity of \( P \)? Justify your answer.

(a) \( A \) has worst-case time complexity \( O(n^2) \).
(b) \( A \) has worst-case time complexity \( O(n^{3/2}) \).
(c) \( A \) has worst-case time complexity \( O(n) \).
(d) \( A \) has worst-case time complexity \( \Theta(n^2) \).
(e) \( A \) has worst-case time complexity \( \Theta(n^3) \).
3. Find the exact solution to the following recursive formulas. You may guess the solution and then prove it by induction.

(a) $T(1) = 1$ and $T(n) = T(n - 1) + 3$.

(b) $T(1) = 1$ and $T(n) = T(n - 1) + (2n - 1)$.

(c) $T(1) = 3$ and $T(n) = 3T(n - 1)$. 
4. Find the exact solution to the following recursive formulas. You may guess the solution and then prove it by induction.

(a) \( T(0) = 1 \) and \( T(n) = 1 + \sum_{i=0}^{n-1} T(i) \).

(b) \( T(0) = 1 \) and \( T(n) = b + \sum_{i=0}^{n-1} T(i) \).

(c) \( T(0) = a \) and \( T(n) = b + \sum_{i=0}^{n-1} T(i) \).
5. Solve the following recursive formulas using the master theorem. Assume that $n = 2^k$ for some integer $k$ for parts (a) and (c) and that $n = (4/3)^k$ for some integer $k$ for part (b).

(a) $T(1) = 1$ and $T(n) = 8T(n/2) + n^2$.

(b) $T(1) = 1$ and $T(n) = T(3n/4) + 10$.

(c) $T(1) = 1$ and $T(n) = T(n/2) + \sqrt{n}$.
6. What value is returned by the following functions? Express your answer as a function of \( n \).

Give the worst-case running time of the following functions using the \( O, \Omega, \Theta \) notations.

(a) \( f(n) \) (* \( n > 0 \) is an integer number *)
\[
\begin{align*}
  r &:= 0 \\
  &\text{for } i := 1 \text{ to } n - 1 \text{ do} \\
  &\quad \text{for } j := i + 1 \text{ to } n \text{ do} \\
  &\quad\quad \text{for } k := 1 \text{ to } j \text{ do} \\
  &\quad\quad\quad r := r + 1 \\
  &\text{return}(r)
\end{align*}
\]

(b) \( f(n) \) (* \( n > 0 \) is an integer number *)
\[
\begin{align*}
  r &:= 0 \\
  &\text{for } i := 1 \text{ to } n \text{ do} \\
  &\quad \text{for } j := 1 \text{ to } i \text{ do} \\
  &\quad\quad \text{for } k := j \text{ to } i + j \text{ do} \\
  &\quad\quad\quad r := r + 1 \\
  &\text{return}(r)
\end{align*}
\]

(c) \( g(x) \) (* \( x > 1 \) is a real number *)
\[
\begin{align*}
  r &:= 0 \\
  &\text{while } x > 1 \text{ do} \\
  &\quad x := x/3 \\
  &\quad r := r + 1 \\
  &\text{return}(r)
\end{align*}
\]
7. An input file contains all the integers from 1 to \( n \) exactly once except one missing integer. The \( n - 1 \) integers may appear in any order. The goal is to find the missing integer. A solution may \textbf{scan} the input only \textbf{once}.

(a) Describe a simple solution that requires \( O(n) \)-bits memory.

(b) Describe a “tricky” solution that requires \( O(\log n) \)-bits memory.