Chapter 5

Forecasting

Introduction

- Forecast – to calculate or estimate something in advance or to predict the future
- Managers are always trying to reduce uncertainty and make better estimates of what will happen in the future when they make decisions
  - E.g. inventory to order, new equipment to purchase, investments to make
- This is the main purpose of forecasting

Some firms (especially small ones) use subjective methods
- Seat-of-the pants methods, intuition, experience, guesswork

There are also several quantitative techniques
- Moving averages, exponential smoothing, trend projections, least squares regression analysis

Regardless of what method is used, the same eight steps are used to make the forecast:

1. Determine the use of the forecast — what objective are we trying to obtain?
2. Select the items or quantities that are to be forecasted
3. Determine the time horizon of the forecast — short term (≤ 1 month), medium term (≤ 1 year) or long term (> 1 year)
4. Select the forecasting model or models
5. Gather the data needed to make the forecast
6. Validate the forecasting model
7. Make the forecast
8. Implement the results
Introduction

- These steps are a systematic way of initiating, designing, and implementing a forecasting system.
- When used regularly over time, data is collected routinely and calculations performed automatically.
- There is seldom one superior forecasting system.
- Different organizations may use different techniques.
- Whatever tool works best for a firm is the one they should use.

Time-Series Models

- Time-series models attempt to predict the future by using historical data.
- Assume what happens in the future is a function of what has happened in the past.
- Look at what happened over a period of time and use a series of past data to make a forecast.
- E.g. use past weekly sales to forecast the next week’s sales.

Forecasting Models

- Common time-series models are:
  - Moving average
  - Exponential smoothing
  - Trend projections
  - Decomposition

- Regression analysis (Chapter 4) is used in trend projections and in one type of decomposition model.
- We emphasize time-series forecast in this chapter.
Causal Models

- **Causal models** use variables or factors that might influence the quantity being forecasted (may also include past data).
- The objective is to build a model with the best statistical relationship ($r^2$, $p$-value, etc) between the variable being forecast and the independent variables.
- E.g. daily sales of a cola drink might depend on factors such as temperature, season, humidity, day of the week, and past sales.
- Regression analysis is the most common technique used in causal modeling.

Qualitative Models

- **Qualitative models** incorporate judgmental or subjective factors into the model – *time-series* and *causal* models relying on quantitative data.
- Useful when subjective factors are thought to be important or when accurate quantitative data is difficult to obtain.
- Common qualitative techniques are:
  - Delphi method
  - Jury of executive opinion
  - Sales force composite
  - Consumer market surveys

Qualitative Models

- **Delphi Method** – an iterative group process where a panel of possibly geographically dispersed experts or respondents provide input to decision makers through questionnaires and surveys.
- **Jury of Executive Opinion** – collects opinions of a small group of high-level managers, possibly using statistical models, to estimate the demand.
- **Sales Force Composite** – individual salespersons estimate the sales in their region and the data is combined at a district or national level to reach an overall forecast.
- **Consumer Market Survey** – input is solicited from customers or potential customers regarding their future purchasing plans.

Scatter Diagrams and Forecasting

- Time-series models are based on past data.
- **Scatter diagrams** are very helpful when forecasting time series data, because they depict the relationship between variables.

![Scatter Diagram Example](image-url)
Wacker Distributors wants to forecast sales for three different products:

### Table 5.1: Annual Sales of Three Products

<table>
<thead>
<tr>
<th>YEAR</th>
<th>TELEVISION SETS</th>
<th>RADIOS</th>
<th>COMPACT DISC PLAYERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
<td>300</td>
<td>110</td>
</tr>
<tr>
<td>2</td>
<td>250</td>
<td>310</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>250</td>
<td>320</td>
<td>120</td>
</tr>
<tr>
<td>4</td>
<td>250</td>
<td>330</td>
<td>140</td>
</tr>
<tr>
<td>5</td>
<td>250</td>
<td>340</td>
<td>170</td>
</tr>
<tr>
<td>6</td>
<td>250</td>
<td>350</td>
<td>150</td>
</tr>
<tr>
<td>7</td>
<td>250</td>
<td>360</td>
<td>160</td>
</tr>
<tr>
<td>8</td>
<td>250</td>
<td>370</td>
<td>190</td>
</tr>
<tr>
<td>9</td>
<td>250</td>
<td>380</td>
<td>200</td>
</tr>
<tr>
<td>10</td>
<td>250</td>
<td>390</td>
<td>190</td>
</tr>
</tbody>
</table>

**Sales appear to be constant over time**

Sales = 250

A good estimate of sales in year 11 is 250 televisions.

**Sales appear to be increasing at a constant rate of 10 radios per year**

Sales = 290 + 10(Year)

A reasonable estimate of sales in year 11 is 400 televisions.

This trend line may not be perfectly accurate because of variation from year to year.

Sales appear to be increasing.

A forecast would probably be a larger figure each year.

**Figure 5.2**
Measures of Forecast Accuracy

- We compare forecasted values with actual values to see how well one model works or to compare models.

  Forecast error = Actual value – Forecast value

- A good forecasting model is one that minimizes the overall errors.

- One measure of accuracy is the mean absolute deviation (MAD)

\[
MAD = \frac{\sum|\text{forecast error}|}{n}
\]

- There are other popular measures of forecast accuracy.

  - The mean squared error

\[
\text{MSE} = \frac{\sum(	ext{error})^2}{n}
\]

  - The mean absolute percent error

\[
\text{MAPE} = \frac{\sum|\text{error}|}{\text{actual}} \times 100\%
\]

- Bias is the average error:

\[
bias = \frac{\sum\text{error}}{n}
\]

- Maybe positive or negative.

- Negative errors can cancel positive errors.
Hospital Days Forecast
Error Example

Ms. Smith forecasted total hospital inpatient days last year. Now that the actual data are known, she is reevaluating her forecasting model. Compute the MAD, MSE, and MAPE for her forecast.

<table>
<thead>
<tr>
<th>Month</th>
<th>Forecast</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>JAN</td>
<td>250</td>
<td>243</td>
</tr>
<tr>
<td>FEB</td>
<td>320</td>
<td>315</td>
</tr>
<tr>
<td>MAR</td>
<td>275</td>
<td>286</td>
</tr>
<tr>
<td>APR</td>
<td>260</td>
<td>256</td>
</tr>
<tr>
<td>MAY</td>
<td>250</td>
<td>241</td>
</tr>
<tr>
<td>JUN</td>
<td>275</td>
<td>298</td>
</tr>
<tr>
<td>JUL</td>
<td>300</td>
<td>292</td>
</tr>
<tr>
<td>AUG</td>
<td>325</td>
<td>333</td>
</tr>
<tr>
<td>SEP</td>
<td>320</td>
<td>326</td>
</tr>
<tr>
<td>OCT</td>
<td>350</td>
<td>378</td>
</tr>
<tr>
<td>NOV</td>
<td>365</td>
<td>382</td>
</tr>
<tr>
<td>DEC</td>
<td>380</td>
<td>396</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Month</th>
<th>Forecast</th>
<th>Actual</th>
<th>Error</th>
<th>Error^2</th>
<th>Error/Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>JAN</td>
<td>250</td>
<td>243</td>
<td>7</td>
<td>49</td>
<td>0.03</td>
</tr>
<tr>
<td>FEB</td>
<td>320</td>
<td>315</td>
<td>5</td>
<td>25</td>
<td>0.02</td>
</tr>
<tr>
<td>MAR</td>
<td>275</td>
<td>286</td>
<td>11</td>
<td>121</td>
<td>0.04</td>
</tr>
<tr>
<td>APR</td>
<td>260</td>
<td>256</td>
<td>4</td>
<td>16</td>
<td>0.02</td>
</tr>
<tr>
<td>MAY</td>
<td>250</td>
<td>241</td>
<td>9</td>
<td>81</td>
<td>0.04</td>
</tr>
<tr>
<td>JUN</td>
<td>275</td>
<td>298</td>
<td>23</td>
<td>529</td>
<td>0.08</td>
</tr>
<tr>
<td>JUL</td>
<td>300</td>
<td>292</td>
<td>8</td>
<td>64</td>
<td>0.03</td>
</tr>
<tr>
<td>AUG</td>
<td>325</td>
<td>333</td>
<td>8</td>
<td>64</td>
<td>0.02</td>
</tr>
<tr>
<td>SEP</td>
<td>320</td>
<td>326</td>
<td>6</td>
<td>36</td>
<td>0.02</td>
</tr>
<tr>
<td>OCT</td>
<td>350</td>
<td>378</td>
<td>28</td>
<td>784</td>
<td>0.07</td>
</tr>
<tr>
<td>NOV</td>
<td>365</td>
<td>382</td>
<td>17</td>
<td>289</td>
<td>0.04</td>
</tr>
<tr>
<td>DEC</td>
<td>380</td>
<td>396</td>
<td>16</td>
<td>256</td>
<td>0.04</td>
</tr>
</tbody>
</table>

SUM AVERAGE 142
MAD = 11.83
MAE = 2314
MAPE = 3.75%

Time-Series Forecasting Models

- A time series is a sequence of evenly spaced events (weekly, monthly, quarterly, etc.)
  - Weekly sales of IBM personal computers
  - Quarterly earnings report of a company
- Time-series forecasts predict the future based solely on the past values of the variable
- Other variables, no matter how potentially valuable, are ignored

Decomposition of a Time-Series

- Analysis of time series means breaking the past data into components and projecting them forward.
- A time series typically has four components
  1. **Trend** ($T$) is the gradual upward or downward movement of the data over time
  2. **Seasonality** ($S$) is a pattern of demand fluctuations above or below trend line that repeats at regular intervals
  3. **Cycles** ($C$) are patterns in annual data that occur every several years
  4. **Random variations** ($R$) are “blips” in the data caused by chance and unusual situations – they follow no discernible pattern
There are two general forms of time-series models:

- **The multiplicative model**
  \[ \text{Demand} = T \times S \times C \times R \]

- **The additive model**
  \[ \text{Demand} = T + S + C + R \]

Models may be combinations of these two forms.

Forecasters often assume the random variations are averaged out over time.

Random errors are often assumed to be normally distributed with a mean of zero.

**Moving Averages**

- We start forecasting time series by assuming that the trend (T), seasonal (S) and cyclical (C) components do not exist.
- **Moving averages** can be used when demand is relatively steady over time.
- The next forecast is the average of the most recent \( n \) data values from the time series.
- This method tends to smooth out short-term irregularities in the data series.

Mathematically:

\[
F_{t+1} = \frac{Y_t + Y_{t-1} + \ldots + Y_{t-n+1}}{n}
\]

where:

- \( F_{t+1} \) = forecast for time period \( t + 1 \)
- \( Y_t \) = actual value in time period \( t \)
- \( n \) = number of periods to average
Wallace Garden Supply wants to forecast the monthly demand for its Storage Shed.

They have collected data for the past year (12 months).

They are using a three-month moving average to forecast the demand ($n = 3$).

They calculated the forecasts to evaluate the accuracy of the model.

The forecast for next January using this technique is 16.

---

### Weighted Moving Averages

Weighted moving averages use weights to put more emphasis on recent periods – more responsive to recent changes. Often used when a trend or other pattern is emerging.

Mathematically:

$$ F_{t+1} = \frac{\sum (Weight \ in \ period \ i)(Actual \ value \ in \ period)}{\sum (Weights)} $$

where

$$ w_i = weight \ for \ the \ i^{th} \ observation $$

---

Wallace Garden Supply decides to try a weighted moving average model to forecast demand for its Storage Shed.

Deciding which weights to use requires some experience and a bit of luck – check MAD.

They decide on the following weighting scheme:

<table>
<thead>
<tr>
<th>WEIGHTS APPLIED</th>
<th>PERIOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Last month</td>
</tr>
<tr>
<td>2</td>
<td>Two months ago</td>
</tr>
<tr>
<td>1</td>
<td>Three months ago</td>
</tr>
</tbody>
</table>

- $w_3 \cdot \text{Sales last month}$
- $w_2 \cdot \text{Sales two months ago}$
- $w_1 \cdot \text{Sales three months ago}$

Sum of the weights.
### Weighted Moving Averages

- Both simple (non-weighted) and weighted averages are effective in smoothing out fluctuations in the demand pattern in order to provide stable estimates.

- Two problems:
  - Increasing the size of $n$ smoothes out fluctuations better but makes the method less sensitive to real changes in the data.
  - Moving averages cannot pick up trends very well – they will always stay within past levels and will not predict a change to a higher or lower level.
Exponential Smoothing

- **Exponential smoothing** is a type of moving average forecasting method that is easy to use and requires little record keeping of data – only the last period’s forecast and actual demand are needed.

- New forecast = Last period’s forecast + $\alpha$ (Last period’s actual demand – Last period’s forecast)

Where $\alpha$ is a weight (or smoothing constant) with a value between 0 and 1 inclusive.

Mathematically:

$$ F_{t+1} = F_t + \alpha(Y_t - F_t) = \alpha Y_t + (1 - \alpha) F_t $$

where

- $F_{t+1}$ = new forecast (for time period $t + 1$)
- $F_t$ = previous forecast (for time period $t$)
- $\alpha$ = smoothing constant ($0 \leq \alpha \leq 1$)
- $Y_t$ = previous period’s actual demand

The idea is simple – the new estimate is the old estimate plus some fraction of the error in the last period.

- $\alpha$ can be increased to give more weight to recent data or decreased to give more weight to past data.

### Exponential Smoothing Example

- In January, February’s demand for a certain car model was predicted to be 142.
- Actual February demand was 153 autos.
- Using a smoothing constant of $\alpha = 0.20$, what is the forecast for March?

New forecast (for March demand) = $142 + 0.2(153 - 142) = 144.2$ or 144 autos.

- If actual demand in March was 136 autos, the April forecast would be

New forecast (for April demand) = $144.2 + 0.2(136 - 144.2) = 142.6$ or 143 autos.

### Selecting the Smoothing Constant

- Selecting the appropriate value for $\alpha$ is key to obtaining a good forecast.
- The objective is always to generate an accurate forecast.
- The general approach is to develop trial forecasts with different values of $\alpha$ and select the $\alpha$ that results in the lowest $MAD$. 

Port of Baltimore Example

Exponential smoothing forecast for two values of $\alpha$

<table>
<thead>
<tr>
<th>QUARTER</th>
<th>ACTUAL TONNAGE UNLOADED</th>
<th>FORECAST USING $\alpha=0.10$</th>
<th>FORECAST USING $\alpha=0.50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>180</td>
<td>175</td>
<td>175</td>
</tr>
<tr>
<td>2</td>
<td>168</td>
<td>175.5 = 175.00 + 0.10(180 – 175)</td>
<td>177.5</td>
</tr>
<tr>
<td>3</td>
<td>159</td>
<td>174.75 = 175.50 + 0.10(168 – 175.50)</td>
<td>172.75</td>
</tr>
<tr>
<td>4</td>
<td>175</td>
<td>173.18 = 174.75 + 0.10(159 – 174.75)</td>
<td>165.88</td>
</tr>
<tr>
<td>5</td>
<td>190</td>
<td>173.36 = 173.18 + 0.10(175 – 173.18)</td>
<td>170.44</td>
</tr>
<tr>
<td>6</td>
<td>205</td>
<td>175.02 = 173.36 + 0.10(190 – 173.36)</td>
<td>180.22</td>
</tr>
<tr>
<td>7</td>
<td>180</td>
<td>175.02 = 175.02 + 0.10(205 – 175.02)</td>
<td>192.61</td>
</tr>
<tr>
<td>8</td>
<td>182</td>
<td>178.22 = 178.02 + 0.10(180 – 178.02)</td>
<td>186.30</td>
</tr>
<tr>
<td>9</td>
<td>?</td>
<td>178.60 = 178.22 + 0.10(182 – 178.22)</td>
<td>184.15</td>
</tr>
</tbody>
</table>

Table 5.5: Assume the initial forecast is 175.

Selecting the Best Value of $\alpha$

<table>
<thead>
<tr>
<th>QUARTER</th>
<th>ACTUAL TONNAGE UNLOADED</th>
<th>FORECAST WITH $\alpha=0.10$</th>
<th>ABSOLUTE DEVIATIONS</th>
<th>FORECAST WITH $\alpha=0.50$</th>
<th>ABSOLUTE DEVIATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>180</td>
<td>175</td>
<td>5</td>
<td>175</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>168</td>
<td>175.5</td>
<td>7.5</td>
<td>177.5</td>
<td>9.5</td>
</tr>
<tr>
<td>3</td>
<td>159</td>
<td>174.75</td>
<td>15.75</td>
<td>172.75</td>
<td>13.75</td>
</tr>
<tr>
<td>4</td>
<td>175</td>
<td>173.18</td>
<td>1.82</td>
<td>165.88</td>
<td>9.12</td>
</tr>
<tr>
<td>5</td>
<td>190</td>
<td>173.36</td>
<td>16.64</td>
<td>170.44</td>
<td>19.56</td>
</tr>
<tr>
<td>6</td>
<td>205</td>
<td>175.02</td>
<td>29.98</td>
<td>180.22</td>
<td>24.78</td>
</tr>
<tr>
<td>7</td>
<td>180</td>
<td>176.02</td>
<td>1.98</td>
<td>192.61</td>
<td>12.61</td>
</tr>
<tr>
<td>8</td>
<td>182</td>
<td>178.22</td>
<td>3.78</td>
<td>186.30</td>
<td>4.3</td>
</tr>
</tbody>
</table>

Table 5.6: Best choice

Program 5.2A

Program 5.2B
PM Computer Example

- PM Computer assembles customized personal computers from generic parts.
- The owners purchase generic computer parts in volume at a discount from a variety of sources whenever they see a good deal.
- It is important that they develop a good forecast of demand for their computers so they can purchase component parts efficiently.

PM Computer Example

<table>
<thead>
<tr>
<th>Period</th>
<th>Month</th>
<th>Actual Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Jan</td>
<td>37</td>
</tr>
<tr>
<td>2</td>
<td>Feb</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>Mar</td>
<td>41</td>
</tr>
<tr>
<td>4</td>
<td>Apr</td>
<td>37</td>
</tr>
<tr>
<td>5</td>
<td>May</td>
<td>45</td>
</tr>
<tr>
<td>6</td>
<td>June</td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>July</td>
<td>43</td>
</tr>
<tr>
<td>8</td>
<td>Aug</td>
<td>47</td>
</tr>
<tr>
<td>9</td>
<td>Sept</td>
<td>56</td>
</tr>
</tbody>
</table>

- **Compute a 2-month moving average**
- **Compute a 3-month weighted moving average using weights of 4,2,1 for the past three months of data**
- **Compute an exponential smoothing forecast using $\alpha = 0.7$, initial forecast of 40**

**Using MAD, what forecast is most accurate?**

<table>
<thead>
<tr>
<th>Actual Demand</th>
<th>2 Month MA</th>
<th>Abs. Dev.</th>
<th>3 Month WMA (4,2,1)</th>
<th>Abs. Dev.</th>
<th>Exp. Sm. ($\alpha=0.7$)</th>
<th>Abs. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>37</td>
<td>40</td>
<td>3.0</td>
<td>38.5</td>
<td>2.1</td>
<td>39.37</td>
<td>1.63</td>
</tr>
<tr>
<td>40</td>
<td>37.9</td>
<td>2.1</td>
<td>40.00</td>
<td>3.0</td>
<td>37.9</td>
<td>2.1</td>
</tr>
<tr>
<td>41</td>
<td>38.50</td>
<td>2.50</td>
<td>40.14</td>
<td>3.14</td>
<td>39.37</td>
<td>1.63</td>
</tr>
<tr>
<td>37</td>
<td>40.50</td>
<td>3.50</td>
<td>40.00</td>
<td>3.0</td>
<td>40.51</td>
<td>3.51</td>
</tr>
<tr>
<td>45</td>
<td>39.00</td>
<td>6.00</td>
<td>38.57</td>
<td>6.43</td>
<td>38.05</td>
<td>6.59</td>
</tr>
<tr>
<td>50</td>
<td>41.00</td>
<td>9.00</td>
<td>42.14</td>
<td>7.86</td>
<td>42.92</td>
<td>7.08</td>
</tr>
<tr>
<td>43</td>
<td>47.50</td>
<td>4.50</td>
<td>46.71</td>
<td>3.71</td>
<td>47.87</td>
<td>4.87</td>
</tr>
<tr>
<td>47</td>
<td>46.50</td>
<td>0.50</td>
<td>45.29</td>
<td>1.71</td>
<td>44.46</td>
<td>2.54</td>
</tr>
<tr>
<td>56</td>
<td>45.00</td>
<td>11.00</td>
<td>46.29</td>
<td>9.71</td>
<td>46.24</td>
<td>9.76</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>37</strong></td>
<td><strong>32.57</strong></td>
<td><strong>46.29</strong></td>
<td><strong>41.45</strong></td>
<td><strong>41.45</strong></td>
<td><strong>4.61</strong></td>
</tr>
<tr>
<td><strong>MAD</strong></td>
<td><strong>5.29</strong></td>
<td><strong>5.43</strong></td>
<td></td>
<td></td>
<td><strong>4.61</strong></td>
<td></td>
</tr>
</tbody>
</table>

Exponential smoothing resulted in the lowest MAD.
Exponential Smoothing with Trend Adjustment

- Like all averaging techniques, simple exponential smoothing does not respond well to trends.
- So a more complex model can be used that adjusts for trends.
- The basic approach is to develop a simple exponential smoothing forecast (unadjusted) then adjust for positive or negative lag in the trend.

Forecast including trend ($FIT_{t+1}$) + Adjusted forecast

= Simple unadjusted forecast ($F_{t+1}$) +
  Trend correction ($T_{t+1}$)

Selecting a Smoothing Constant

- As with simple exponential smoothing, a high value of $\beta$ makes the forecast more responsive to recent changes in trend.
- A low value of $\beta$ gives less weight to the recent trend and tends to smooth out the trend present.
- Values of $\beta$ are generally selected using a trial-and-error approach with the MAD used as a measure of comparison.
- Simple exponential smoothing is often referred to as first-order smoothing.
- Trend-adjusted smoothing is called second-order, double smoothing, or Holt's method.

Exponential Smoothing with Trend Adjustment Example

- An electronics company is selling portable CD players and needed to forecast the next three periods’ demands using the Adjusted Exponential Smoothing model.
- To start the forecast an initial estimate of 50 players was used for the first period.
- The actual demand for the first three periods are 54, 57 and 44 CD players.
- $\beta = 0.7$ and $\alpha = 0.2$.
- See Table 1.

The equation for the trend correction uses a new smoothing constant $\beta$, in a way similar to the simple exponential model uses $\alpha$.

$T_{t+1}$ is computed by

$$T_{t+1} = (1 - \beta)T_t + \beta(F_{t+1} - F_t)$$

where

- $T_{t+1}$ = smoothed trend for period $t + 1$
- $T_t$ = smoothed trend for preceding period
- $\beta$ = trend smooth constant that we select
- $F_{t+1}$ = simple exponential smoothed forecast for period $t + 1$
- $F_t$ = simple exponential smoothed forecast for previous period.
**Exponential Smoothing with Trend Adjustment Example**

<table>
<thead>
<tr>
<th>Period</th>
<th>Demand $Y_t$</th>
<th>Unadjusted Forecast $F_t$</th>
<th>Trend $T_t$</th>
<th>Adjusted Forecast $FIT_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>54</td>
<td>50</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>57</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>44</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Equations for calculating the adjusted forecasts:

\[ F_{t+1} = F_t + \alpha(Y_t - F_t) = \alpha Y_t + (1 - \alpha) F_t \]

\[ T_{t+1} = (1 - \beta) T_t + \beta (F_{t+1} - F_t) \]

\[ FIT_{t+1} = F_{t+1} + T_{t+1} \]

Step 1

- Create a table and enter the figures for the first period.
- Demand was 54.
- Unadjusted Forecast for period 1 is any reasonable starting figure to start the process, in this case 50 players.
- The initial trend is estimated to be 0.

<table>
<thead>
<tr>
<th>Period</th>
<th>Demand $Y_t$</th>
<th>Unadjusted Forecast $F_t$</th>
<th>Trend $T_t$</th>
<th>Adjusted Forecast $FIT_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>54</td>
<td>50</td>
<td>0</td>
<td>50</td>
</tr>
</tbody>
</table>

**Step 2**

Calculate $F_{t+1}$ for period 2:

\[ F_{t+1} = F_t + \alpha(Y_t - F_t) = \alpha Y_t + (1 - \alpha) F_t \]

\[ F_2 = 50 + 0.2(54 - 50) = 50.8 \]

Calculate $FIT_{t+1}$ for period 1:

\[ FIT_{t+1} = F_{t+1} + T_{t+1} \]

<table>
<thead>
<tr>
<th>Period</th>
<th>Demand $Y_t$</th>
<th>Unadjusted Forecast $F_t$</th>
<th>Trend $T_t$</th>
<th>Adjusted Forecast $FIT_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>54</td>
<td>50</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>57</td>
<td>50.8</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Step 3

The trend adjustment factor for period 2 is calculated as follows:

\[
T_{t+1} = (1 - \beta)T_t + \beta(F_{t+1} - F_t)
\]

\[
T_2 = (1 - 0.7) \times 0 + 0.7 \times (50.8 - 50) = 0.56
\]

<table>
<thead>
<tr>
<th>Period</th>
<th>Demand Yt</th>
<th>Unadjusted Forecast Ft</th>
<th>Trend Tt</th>
<th>Adjusted Forecast FITt</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>54</td>
<td>50</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>57</td>
<td>50.8</td>
<td>0.56</td>
<td>51.36</td>
</tr>
</tbody>
</table>

Steps 1-4 Completed

\[
F_3 = 50.8 + 0.2 \times (57 - 50.8) = 52.04
\]

\[
T_3 = (1 - 0.7) \times 0.56 + 0.7 \times (52.04 - 50.8) = 1.036
\]

\[
FIT_3 = F_3 + T_3 = 53.076
\]

<table>
<thead>
<tr>
<th>Period</th>
<th>Demand Yt</th>
<th>Unadjusted Forecast Ft</th>
<th>Trend Tt</th>
<th>Adjusted Forecast FITt</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>54</td>
<td>50</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>57</td>
<td>50.8</td>
<td>0.56</td>
<td>51.36</td>
</tr>
<tr>
<td>3</td>
<td>44</td>
<td>52.04</td>
<td>1.036</td>
<td>53.076</td>
</tr>
</tbody>
</table>

Complete the Table

Now calculate the Adjusted Forecast for period 3.
Another method for forecasting time series with trend is trend projection.

Trend projection fits a trend line to a series of historical data points.

The line is projected into the future for medium- to long-range forecasts.

Several trend equations can be developed (e.g. exponential or quadratic models).

The simplest is a linear model developed using regression analysis (Chp. 4) in which the independent variable ($X$) is the time period.

The mathematical form is:

$$\hat{Y} = b_0 + b_1X$$

where

- $\hat{Y}$ = predicted value
- $b_0$ = intercept
- $b_1$ = slope of the line
- $X$ = time period (i.e., $X = 1, 2, 3, \ldots, n$)

The regression line that minimizes the sum of squared errors (SSE) is used (Fig. 5.4).

Midwestern Manufacturing Company Example

Midwestern Manufacturing Company has experienced the following demand for its electrical generators over the period of 2001 – 2007:

<table>
<thead>
<tr>
<th>YEAR</th>
<th>ELECTRICAL GENERATORS SOLD</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>74</td>
</tr>
<tr>
<td>2002</td>
<td>79</td>
</tr>
<tr>
<td>2003</td>
<td>80</td>
</tr>
<tr>
<td>2004</td>
<td>90</td>
</tr>
<tr>
<td>2005</td>
<td>105</td>
</tr>
<tr>
<td>2006</td>
<td>142</td>
</tr>
<tr>
<td>2007</td>
<td>122</td>
</tr>
</tbody>
</table>

Table 5.7
The forecast equation is

\[ \hat{Y} = 56.71 + 10.54X \]

To project demand for 2008, we use the coding system to define \( X = 8 \)

\[
\text{sales in 2008} = 56.71 + 10.54(8) = 141.03 \text{, or 141 generators}
\]

Likewise for \( X = 9 \)

\[
\text{sales in 2009} = 56.71 + 10.54(9) = 151.57 \text{, or 152 generators}
\]
Now we consider forecasting time series with seasonal component.

Recurring variations over time may indicate the need for seasonal adjustments in the trend line.

A seasonal index indicates how a particular season compares with an average season.

When no trend is present, the seasonal index can be found by dividing the average value for a particular season by the average of all the data.
Seasonal Variations

- Eichler Supplies sells telephone answering machines.
- Data has been collected for the past two years’ sales of one particular model.
- They want to create a forecast that includes seasonality.

<table>
<thead>
<tr>
<th>MONTH</th>
<th>SALES DEMAND YEAR 1</th>
<th>SALES DEMAND YEAR 2</th>
<th>AVERAGE TWO-YEAR DEMAND</th>
<th>AVERAGE MONTHLY DEMAND</th>
<th>AVERAGE SEASONAL INDEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>80</td>
<td>100</td>
<td>90</td>
<td>94</td>
<td>0.957</td>
</tr>
<tr>
<td>February</td>
<td>85</td>
<td>75</td>
<td>80</td>
<td>94</td>
<td>0.851</td>
</tr>
<tr>
<td>March</td>
<td>80</td>
<td>90</td>
<td>85</td>
<td>94</td>
<td>0.904</td>
</tr>
<tr>
<td>April</td>
<td>110</td>
<td>90</td>
<td>100</td>
<td>94</td>
<td>1.064</td>
</tr>
<tr>
<td>May</td>
<td>115</td>
<td>131</td>
<td>123</td>
<td>94</td>
<td>1.309</td>
</tr>
<tr>
<td>June</td>
<td>120</td>
<td>110</td>
<td>115</td>
<td>94</td>
<td>1.223</td>
</tr>
<tr>
<td>July</td>
<td>100</td>
<td>110</td>
<td>105</td>
<td>94</td>
<td>1.117</td>
</tr>
<tr>
<td>August</td>
<td>110</td>
<td>90</td>
<td>100</td>
<td>94</td>
<td>1.064</td>
</tr>
<tr>
<td>September</td>
<td>85</td>
<td>95</td>
<td>90</td>
<td>94</td>
<td>0.957</td>
</tr>
<tr>
<td>October</td>
<td>75</td>
<td>85</td>
<td>80</td>
<td>94</td>
<td>0.851</td>
</tr>
<tr>
<td>November</td>
<td>85</td>
<td>75</td>
<td>80</td>
<td>94</td>
<td>0.851</td>
</tr>
<tr>
<td>December</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>94</td>
<td>0.851</td>
</tr>
</tbody>
</table>

Total average demand = 1,128

Average monthly demand = \( \frac{1,128}{12 \text{ months}} = 94 \)

Seasonal index = \( \frac{\text{Average two-year demand}}{\text{Average monthly demand}} \)

Table 5.8

Seasonal Variations with Trend

- Suppose the third year’s annual demand is expected to be 1200 units (average 100 units/month).
- We forecast each month’s demand by adjusting the monthly average based on the seasonal indices:

  Jan. 100 \times 0.957 = 96
  Feb. 100 \times 0.851 = 85
  Mar. 100 \times 0.904 = 90
  Apr. 100 \times 1.064 = 106
  May 100 \times 1.309 = 131
  June 100 \times 1.223 = 122
  July 100 \times 1.117 = 112
  Aug. 100 \times 1.064 = 106
  Sept. 100 \times 0.957 = 96
  Oct. 100 \times 0.851 = 85
  Nov. 100 \times 0.851 = 85
  Dec. 100 \times 0.851 = 85

- When both trend and seasonal components are present, the forecasting task is more complex.
- Changes in time series may be due to a trend, to a seasonal variation or simply random fluctuations.
- Seasonal indices should be computed relative to the trend not the overall average.
- Centered moving average (CMA) is an approximation of the trend (see Fig. 5.6) and is used to find the seasonal indices.
**Turner Industries Example**

- The following are Turner Industries’ sales figures for the past three years

<table>
<thead>
<tr>
<th>QUARTER</th>
<th>YEAR 1</th>
<th>YEAR 2</th>
<th>YEAR 3</th>
<th>AVERAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>108</td>
<td>116</td>
<td>123</td>
<td>115.67</td>
</tr>
<tr>
<td>2</td>
<td>125</td>
<td>134</td>
<td>142</td>
<td>133.67</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>159</td>
<td>168</td>
<td>159.00</td>
</tr>
<tr>
<td>4</td>
<td>141</td>
<td>152</td>
<td>165</td>
<td>152.67</td>
</tr>
<tr>
<td>Average</td>
<td>131.00</td>
<td>140.25</td>
<td>149.50</td>
<td>140.25</td>
</tr>
</tbody>
</table>

Table 5.9

**Definite trend**  
**Seasonal pattern**

**Seasonal Variations with Trend**

- There are four steps in computing seasonal indices using CMA method
  1. Compute the CMA for each observation (where possible)
  2. Compute the seasonal ratio = \( \text{Observation/CMA for that observation} \)
  3. Average seasonal ratios to get seasonal indices
  4. If seasonal indices do not add to the number of seasons, multiply each index by: \( \frac{\text{Number of seasons}}{\text{Sum of indices}} \)

**Turner Industries Example**

- Scatter plot of Turner Industries data shows seasonality component

- To calculate the CMA for quarter 3 of year 1 we compare the actual sales with an average quarter centered on that time period

  We will use a **one year** CMA – 1.5 quarters before quarter 3 and 1.5 quarters after quarter 3 – that is we take quarters 2, 3, and 4 and one half of quarters 1, year 1 and quarter 1, year 2

  \[
  \text{CMA}(\text{q3, y1}) = \frac{0.5(108) + 125 + 150 + 141 + 0.5(116)}{4} = 132.00
  \]

  \[
  \text{CMA}(\text{q2, y3}) = \frac{0.5(152) + 123 + 142 + 168 + 0.5(165)}{4} = 147.875
  \]
We compare the actual sales in Y1Q3 to the CMA to find the seasonal ratio

\[
\text{Seasonal ratio} = \frac{\text{Sales in Y1Q3}}{\text{CMA}} = \frac{150}{132} = 1.136
\]

We see that sales in Y1Q3 are about 13.6% higher than an average quarter at this time.

For Y3Q2, the sales are about 4% lower than an average quarter centered at that time.

\[
\text{Seasonal ratio} = \frac{\text{Sales in Y3Q2}}{\text{CMA}} = \frac{142}{147.875} = 0.960
\]

There are two seasonal ratios for each quarter so these are averaged to get the seasonal index.

Index for quarter 1 = \( I_1 = (0.851 + 0.848)/2 = 0.85 \)
Index for quarter 2 = \( I_2 = (0.965 + 0.960)/2 = 0.96 \)
Index for quarter 3 = \( I_3 = (1.136 + 1.127)/2 = 1.13 \)
Index for quarter 4 = \( I_4 = (1.051 + 1.063)/2 = 1.06 \)

Notice: the sum of the indices should be 4.

\( (0.85+0.96+1.13+1.06 = 4) \)

If not, an adjustment would be made.

Scatter plot of Turner Industries data and CMAs

CMA plot approximates the trend line.
The Decomposition Method of Forecasting

- **Decomposition** (or *multiplicative decomposition*) is the process of isolating 1) the seasonal factors and 2) the linear trend line to develop more accurate forecasts.

- There are five steps to decomposition:
  1. Compute CMAs and seasonal indices.
  2. Deseasonalize the data by dividing each number by its seasonal index (deseasonalized data approximates the trend line).
  3. Find the regression equation of a trend line using the deseasonalized data.
  4. Forecast for future periods using the trend line.
  5. Multiply the trend line forecast by the appropriate seasonal index.

### Turner Industries – Decomposition Method

<table>
<thead>
<tr>
<th>YEAR</th>
<th>QUARTER</th>
<th>SALES ($1,000,000s)</th>
<th>SEASONAL INDEX</th>
<th>DESEASONALIZED SALES ($1,000,000s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>108</td>
<td>0.85</td>
<td>127.059</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>125</td>
<td>0.96</td>
<td>130.208</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>150</td>
<td>1.13</td>
<td>132.743</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>141</td>
<td>1.06</td>
<td>133.019</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>116</td>
<td>0.85</td>
<td>136.471</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>134</td>
<td>0.96</td>
<td>139.583</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>159</td>
<td>1.13</td>
<td>140.708</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>152</td>
<td>1.06</td>
<td>143.396</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>123</td>
<td>0.85</td>
<td>144.706</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>142</td>
<td>0.96</td>
<td>147.917</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>168</td>
<td>1.13</td>
<td>148.673</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>165</td>
<td>1.06</td>
<td>155.660</td>
</tr>
</tbody>
</table>

Table 5.11

### Turner Industries Example

- Using computer software to find a trend line using the deseasonalized data:
  - $b_0 = 124.78$
  - $b_1 = 2.34$

- Develop a forecast for the first quarter of year four ($X = 13$) using this trend and multiply the forecast by the appropriate seasonal index ($I_1 = 0.85$):
  - $\hat{Y} = 124.78 + 2.34X$
  - $\hat{Y} = 124.78 + 2.34(13)$
  - $\hat{Y} = 155.2$ (forecast before adjustment for seasonality)

- $\hat{Y} \times I_1 = 155.2 \times 0.85 = 131.92$
A San Diego hospital developed a decomposition forecasting model using 66 months of adult inpatient hospital days. They developed the following seasonal indices:

<table>
<thead>
<tr>
<th>MONTH</th>
<th>SEASONALITY INDEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1.0436</td>
</tr>
<tr>
<td>February</td>
<td>0.9669</td>
</tr>
<tr>
<td>March</td>
<td>1.0203</td>
</tr>
<tr>
<td>April</td>
<td>1.0087</td>
</tr>
<tr>
<td>May</td>
<td>0.9935</td>
</tr>
<tr>
<td>June</td>
<td>0.9906</td>
</tr>
<tr>
<td>July</td>
<td>1.0302</td>
</tr>
<tr>
<td>August</td>
<td>1.0405</td>
</tr>
<tr>
<td>September</td>
<td>0.9653</td>
</tr>
<tr>
<td>October</td>
<td>1.0048</td>
</tr>
<tr>
<td>November</td>
<td>0.9598</td>
</tr>
<tr>
<td>December</td>
<td>0.9805</td>
</tr>
</tbody>
</table>

Table 5.12

Using the data they developed the following equation for the trend line:

\[ \hat{Y} = 8,091 + 21.5X \]

where

\[ \hat{Y} = \text{forecast of patient days} \]

\[ X = \text{time period in months} \]

Based on this model, the forecast for patient days for the next month (period 67 and January) is

Patient days = 8,091 + (21.5)(67) = 9,532 (trend only)

Patient days = (9,532)(1.0436) = 9,948 (trend and seasonal)

Multiple regression (or additive decomposition model) can be used to forecast both trend and seasonal components in a time series:

- One independent variable is time period
- Dummy independent variables are used to represent the seasons – for 4 seasons 3 dummy variables are used

\[ \hat{Y} = a + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 \]

where

\[ X_1 = \text{time period} \]

\[ X_2 = 1 \text{ if quarter 2, 0 otherwise} \]

\[ X_3 = 1 \text{ if quarter 3, 0 otherwise} \]

\[ X_4 = 1 \text{ if quarter 4, 0 otherwise} \]

If \( X_2 = X_3 = X_4 = 0 \), then the quarter would be quarter 1.

Which \( X_1 \) is used for which season is chosen arbitrarily.
The resulting regression equation is

\[ \hat{Y} = 104.1 + 2.3X_1 + 15.7X_2 + 38.7X_3 + 30.1X_4 \]

Using the model to forecast sales for the first and second quarter of next year

\[ \hat{Y} = 104.1 + 2.3(13) + 15.7(0) + 38.7(0) + 30.1(0) = 134 \]

\[ \hat{Y} = 104.1 + 2.3(14) + 15.7(1) + 38.7(0) + 30.1(0) = 152 \]

These are different from the results obtained using the multiplicative decomposition method. Use MAD and MSE to determine the best model.

Tracking signals can be used to monitor the performance of a forecast.

Tracking signals are computed as the running sum of the forecast errors divided by the mean absolute deviation.

\[ \text{Tracking signal} = \frac{\text{RSFE}}{\text{MAD}} = \frac{\sum (\text{forecast error})}{\text{MAD}} \]

where

\[ \text{MAD} = \frac{1}{n} \sum |\text{forecast error}| \]
Monitoring and Controlling Forecasts

- Positive tracking signals indicate demand is greater than forecast
- Negative tracking signals indicate demand is less than forecast
- Some variation is expected, but a good forecast will have about as much positive error as negative error
- Problems are indicated when the signal trips either the upper or lower predetermined limits
- This indicates there has been an unacceptable amount of variation
- Limits should be reasonable and may vary from item to item (one suggested limits = ±4 MADs)

Kimball’s Bakery Example

- Tracking signal for quarterly sales of croissants

<table>
<thead>
<tr>
<th>TIME PERIOD</th>
<th>FORECAST DEMAND</th>
<th>ACTUAL DEMAND</th>
<th>FORECAST ERROR</th>
<th>RSFE</th>
<th>FORECAST ERROR</th>
<th>CUMULATIVE ERROR</th>
<th>MAD</th>
<th>TRACKING SIGNAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>90</td>
<td>–10</td>
<td>–10</td>
<td>10</td>
<td>10.0</td>
<td>10</td>
<td>–1</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>95</td>
<td>–5</td>
<td>–15</td>
<td>5</td>
<td>7.5</td>
<td>15</td>
<td>–2</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>115</td>
<td>+15</td>
<td>0</td>
<td>15</td>
<td>10.0</td>
<td>30</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>110</td>
<td>100</td>
<td>–10</td>
<td>–10</td>
<td>10</td>
<td>10.0</td>
<td>40</td>
<td>–1</td>
</tr>
<tr>
<td>5</td>
<td>110</td>
<td>125</td>
<td>+15</td>
<td>+5</td>
<td>15</td>
<td>11.0</td>
<td>55</td>
<td>+0.5</td>
</tr>
<tr>
<td>6</td>
<td>110</td>
<td>140</td>
<td>+30</td>
<td>+35</td>
<td>30</td>
<td>14.2</td>
<td>85</td>
<td>+2.5</td>
</tr>
</tbody>
</table>

For period 6:

\[
\text{MAD} = \frac{\sum |\text{forecast error}|}{n} = \frac{85}{6} = 14.2
\]

\[
\text{Tracking signal} = \frac{\text{RSFE}}{\text{MAD}} = \frac{35}{14.2} = 2.5 \text{MADs}
\]

Adaptive Smoothing

- *Adaptive smoothing* is the computer monitoring of tracking signals and self-adjustment if a limit is tripped
- In exponential smoothing, the values of \( \alpha \) and \( \beta \) are adjusted when the computer detects an excessive amount of variation
Using The Computer to Forecast

- Spreadsheets (*Excel* and *Excel QM*) can be used by small and medium-sized forecasting problems.
- More advanced programs (*SAS*, *SPSS*, *Minitab*) handle time-series and causal models.
- May automatically select best model parameters.
- Dedicated forecasting packages may be fully automatic.
- May be integrated with inventory planning and control.

Homework Assignment

[http://www.sci.brooklyn.cuny.edu/~dzhu/busn3430/](http://www.sci.brooklyn.cuny.edu/~dzhu/busn3430/)