Inventory is any stored resource used to satisfy a current or future need. Common examples are raw materials, work-in-process, and finished goods. Inventory is an expensive and important asset to many companies – up to 50% of the total invested capital. Lower inventory levels can reduce costs. Low inventory levels may result in stockouts and dissatisfied customers. Most companies try to balance high and low inventory levels with cost minimization as a goal.

Inventory may account for 50% of the total invested capital of an organization and 70% of the cost of goods sold. All organizations have some type of inventory planning and control system. E.g.1: a bank has methods to control its cash. E.g.2: a hospital has methods to control blood supplies. The study of organizations’ inventory control requires the understanding of how they supply goods and services to the customers.
Introduction

- Basic components of inventory planning
  - Planning what inventory is to be stocked and how it is to be acquired (purchased or manufactured)
  - This information is used in forecasting demand for the inventory and in controlling inventory levels
  - Feedback provides a means to revise the plan and forecast based on experiences and observations

Introduction

- Basic components of an inventory planning and control system

  - Planning on What Inventory to Stock and How to Acquire It
  - Forecasting Parts/Product Demand
  - Controlling Inventory Levels

  - Feedback Measurements to Revise Plans and Forecasts

Figure 6.1

Introduction

- Inventory planning helps determine what goods and/or services need to be produced
- Inventory planning helps determine whether the organization produces the goods or services or whether they are purchased from another organization
- Inventory planning also involves demand forecasting
Importance of Inventory Control

- Five uses of inventory
  - The decoupling function
  - Storing resources
  - Irregular supply and demand
  - Quantity discounts
  - Avoiding stockouts and shortages

- The decoupling function
  - Used as a buffer between stages in a manufacturing process
  - Reduces delays and improves efficiency
  - e.g. one activity has to be completed before the next activity can be started

- Storing resources
  - Seasonal products may be stored to satisfy off-season demand
  - Materials can be stored as raw materials, work-in-process, or finished goods
  - Labor can be stored as a component of partially completed subassemblies

- Irregular supply and demand
  - Demand and supply may not be constant over time
  - Inventory can be used to buffer the variability

Importance of Inventory Control

- Quantity discounts
  - Many suppliers offer discounts for large orders
  - Extra costs associated with holding more inventory must be balanced against lower purchase price
    - Higher storage costs and other higher costs due to damage, theft, insurance, etc.

- Avoiding stockouts and shortages
  - Stockouts may result in lost sales
  - Dissatisfied customers may choose to buy from another supplier

Inventory Decisions

- There are only two fundamental decisions in controlling inventory of any product
  - How much to order
  - When to order

- The major objective of all inventory models is to minimize total inventory costs
### Inventory Cost Factors

- Common inventory costs are
  - Cost of the items (purchase or material cost – what is paid to acquire the inventory)
  - Cost of ordering
  - Cost of carrying, or holding, inventory
  - Cost of stockouts (lost sales and goodwill or future sales that result from not having the items available for the customers)

### Ordering Cost Factors

<table>
<thead>
<tr>
<th>ORDERING COST FACTORS</th>
<th>CARRYING COST FACTORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Developing and sending purchase orders</td>
<td>Cost of capital</td>
</tr>
<tr>
<td>Processing and inspecting incoming inventory</td>
<td>Taxes</td>
</tr>
<tr>
<td>Bill paying</td>
<td>Insurance</td>
</tr>
<tr>
<td>Inventory inquiries</td>
<td>Spoilage</td>
</tr>
<tr>
<td>Utilities, phone bills, and so on, for the purchasing department</td>
<td>Theft</td>
</tr>
<tr>
<td>Salaries and wages for the purchasing department employees</td>
<td>Obsolescence</td>
</tr>
<tr>
<td>Supplies such as forms and paper for the purchasing department</td>
<td>Salaries and wages for warehouse employees</td>
</tr>
<tr>
<td>Utilities and building costs for the warehouse</td>
<td>Supplies such as forms and paper for the warehouse</td>
</tr>
</tbody>
</table>

- Ordering costs are generally independent of order quantity – incurred for each order
  - Many involve personnel time – paperwork, bill...
  - The amount of work is the same for each order no matter the size of the order
- Carrying costs generally varies with the amount of inventory, or the order size
  - The labor, space, and other costs increase as the order size increases

### Economic Order Quantity

- The economic order quantity (EOQ) model is one of the oldest and most commonly known inventory control techniques.
- It dates from 1915.
- It is still used by a large number of organizations today.
- It is relatively easy to use but it makes a number of important assumptions.
Some EOQ Assumptions

1. Demand is known and constant
2. Lead time (the time between the placement and receipt of an order) is known and constant
3. Receipt of inventory is instantaneous (inventory from an order arrives in one batch, at one point in time)
4. Purchase cost per unit is constant throughout the year and there are no quantity discounts
5. The only variable costs are ordering cost for placing orders and holding or carrying cost for holding or storing inventory over time, and ordering cost per order and holding cost per unit per year are constant throughout the year
6. Orders are placed so that stockouts or shortages are avoided completely

If previous assumptions are not met, adjustments must be made to the EOQ model

Given the previous assumptions, inventory usage has a sawtooth shape (Fig. 6.2)

Inventory jumps from 0 to the maximum when the shipment arrives

Because demand is constant over time, inventory drops at a uniform rate over time

This process continues indefinitely over time

Objective is generally to minimize total cost

The relevant costs are ordering costs and carrying costs and all other costs can be ignored – the purchase costs are constant and no stockout costs

Thus if we minimize the sum of the ordering and carrying costs, we also minimize the total costs

The annual ordering cost is the number of orders per year times the cost of placing each order

As the inventory level changes daily, we use the average inventory level to determine annual holding or carrying cost

The annual carrying cost equals the average inventory times the inventory carrying cost per unit per year

The maximum inventory is Q and the average inventory is \( Q/2 \)

Inventory Usage Over Time

Order Quantity = \( Q = \) Maximum Inventory Level

Inventory Costs in the EOQ Situation

Figure 6.2
**Inventory Costs in the EOQ Situation**

Average inventory level = \( \frac{Q}{2} \)

<table>
<thead>
<tr>
<th>DAY</th>
<th>BEGINNING</th>
<th>ENDING</th>
<th>AVERAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 1 (order received)</td>
<td>10</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>April 2</td>
<td>8</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>April 3</td>
<td>6</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>April 4</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>April 5</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Maximum level April 1 = 10 units
Total of daily averages = 9 + 7 + 5 + 3 + 1 = 25
Number of days = 5
Average inventory level = 25/5 = 5 units \( \text{(or } \frac{Q}{2} = \frac{10}{2} = 5) \)

Table 6.2

---

**Inventory Costs in the EOQ Situation**

Mathematical equations for annual ordering cost can be developed using the following variables

- \( Q \) = number of pieces to order
- \( EOQ = Q^* \) = optimal number of pieces to order
- \( D \) = annual demand in units for the inventory item
- \( C_o \) = ordering cost of each order
- \( C_h \) = holding or carrying cost per unit per year

Annual ordering cost = \( \left( \frac{\text{Number of orders placed per year}}{\text{Ordering cost per order}} \right) \times D \times \frac{C_o}{Q} \)

**Inventory Costs in the EOQ Situation**

Mathematical equations for annual holding cost can be developed using the following variables

- \( Q \) = number of pieces to order
- \( EOQ = Q^* \) = optimal number of pieces to order
- \( D \) = annual demand in units for the inventory item
- \( C_o \) = ordering cost of each order
- \( C_h \) = holding or carrying cost per unit per year

Annual holding cost = \( \left( \frac{\text{Average inventory}}{2} \right) \times \left( \frac{\text{Carrying cost per unit per year}}{Q} \right) \times \frac{Q}{2} \times C_h \)

**Total relevant inventory cost** =
Annual ordering cost + Annual holding cost

\( = \frac{D}{Q} C_o + \frac{Q}{2} C_h \)
**Inventory Costs in the EOQ Situation**

![Graph showing the minimum total cost curve and optimal order quantity](image)

**Finding the EOQ**

- When the EOQ assumptions are met, total cost is minimized when \( \text{Annual Ordering Cost} = \text{Annual Holding Cost} \)

\[
\frac{D}{Q} C_o = \frac{Q}{2} C_h
\]

- Solving for \( Q \)

\[
\frac{2DC_o}{C_h} = Q^2
\]

\[
Q = \text{EOQ} = Q^* = \sqrt{\frac{2DC_o}{C_h}}
\]

**Economic Order Quantity (EOQ) Model**

- Summary of equations

\[
\text{Annual ordering cost} = \frac{D}{Q} C_o
\]

\[
\text{Annual holding cost} = \frac{Q}{2} C_h
\]

\[
\text{EOQ} = Q^* = \sqrt{\frac{2DC_o}{C_h}}
\]

**Sumco Pump Company Example**

- Company sells pump housings to other companies
- Would like to reduce inventory costs by finding optimal order quantity
- Annual demand = 1,000 units
- Ordering cost = $10 per order
- Average carrying cost per unit per year = $0.50
- Meets all EOQ assumptions

\[
Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2(1,000)(10)}{0.50}} = \sqrt{40,000} = 200 \text{ units}
\]
The **relevant** total annual inventory cost is calculated as:

\[
TC = \frac{D}{Q} C_o + \frac{Q}{2} C_h
\]

\[
= \frac{1,000}{200} (10) + \frac{200}{2} (0.5)
\]

\[
= 50 + 50 = 100
\]

The **actual** total inventory cost should also include the purchase cost.

---

**Purchase Cost of Inventory Items**

- Total inventory cost can be written to include the cost of purchased items:
  - Total cost = Order cost + Holding cost + Purchase cost
- Given the EOQ assumptions, the annual purchase cost is constant at \( D \times C \) no matter the order policy:
  - \( C \) is the purchase cost per unit
  - \( D \) is the annual demand in units
- It may be useful to know the average dollar level of inventory:

\[
\text{Average dollar level} = \frac{(CQ)}{2}
\]
Carrying Cost in Terms of Percentage of Unit Cost

- Inventory carrying cost is often expressed as an annual percentage of the unit cost or price of the inventory.
- This requires a new variable $I$ to be introduced.

$$I = \left( \frac{\text{Annual inventory holding charge as a percentage of unit price or cost}}{\text{unit price or cost}} \right)$$

- The cost of storing one unit of inventory for one year is then

$$C_h = IC$$

thus,

$$Q^* = \sqrt{\frac{2DC}{IC}}$$

Pacific Woods Example

- Company sells plywood.
- Manages its inventory using EOQ model.
  - Ordering cost = $25 per order.
  - Carrying cost = 25% of the unit cost.
  - Purchase cost = $100 per load.
- EOQ = 4000 loads.
- What is the annual demand in loads of plywood?

$$Q^* = \sqrt{\frac{2DC}{IC}}$$

$$Q^* = \sqrt{\frac{2D(25)}{0.25 \times 100}} = \sqrt{\frac{50D}{25}} = \sqrt{2D}$$

$$D = 8000000$$

Sensitivity Analysis with the EOQ Model

- The EOQ model assumes all values are known and fixed over time.
- Generally, however, the values are estimated or may change over time.
- So it is important to understand how the changes will affect the optimal order quantity (EOQ).
- Determining the effects of these changes is called sensitivity analysis.
- Because of the square root in the formula, changes in the inputs result in relatively small changes in the optimal order quantity.

$$\text{EOQ} = \sqrt{\frac{2DC}{C_h}}$$

Sensitivity Analysis with the EOQ Model

- In the Sumco example

$$\text{EOQ} = \sqrt{\frac{2(1000)(10)}{0.50}} = 200 \text{ units}$$

- If the ordering cost were increased four times from $10 to $40, the order quantity would only double.

$$\text{EOQ} = \sqrt{\frac{2(1000)(40)}{0.50}} = 400 \text{ units}$$

- In general, the EOQ changes by the square root of a change to any of the inputs.
Reorder Point: Determining When To Order

Once the order quantity is determined, the next decision is **when to order**.

The time between placing an order and its receipt is called the **lead time (L)** or **delivery time**—a few days or a few weeks.

Inventory must be available during this period to meet the demand.

When to order is generally expressed as a **reorder point (ROP)**—the inventory level at which an order should be placed.

If an order is placed when the inventory level reaches the ROP, the new inventory arrives at the same instant the inventory is reaching 0.

\[ ROP = \left( \text{Demand per day} \right) \times \left( \text{Lead time for a new order in days} \right) = d \times L \]

**Procomp’s Computer Chip Example**

- Demand for the computer chip is 8,000 per year.
- Daily demand is 40 units.
- Delivery takes three working days.
- What is the reorder point?

\[ ROP = d \times L = 40 \text{ units per day} \times 3 \text{ days} = 120 \text{ units} \]

An order is placed when the inventory reaches 120 units.

The order arrives 3 days later just as the inventory is depleted.

**EOQ Without The Instantaneous Receipt Assumption**

- When inventory accumulates over time, the **instantaneous receipt** assumption does not apply.
- Inventory continuously flows or builds up over a period of time after an order has been placed or when units are produced and sold simultaneously.
- The daily demand rate must be considered.
- The revised model is especially suited to the production environment so it is commonly called the **production run model**.
The production run model eliminates the instantaneous receipt assumption. The maximum inventory will be less than the production (or order) quantity $Q$ (why?).

In production runs, if the situation involves production rather than ordering, the ordering cost is replaced by setup cost - cost of setting up the production facility to manufacture the desired product. Including the cost of labor, paperwork, materials, utilities, etc.

The optimal production quantity is derived by setting setup costs equal to holding or carrying costs and solving for the order quantity.

Setup cost replaces ordering cost when a product is produced over time.

The model uses the following variables:

- $Q =$ number of pieces per production run, or order
- $C_s =$ setup cost (or $C_o$ for ordering cost)
- $C_h =$ holding or carrying cost per unit per year
- $p =$ daily production or receipt rate
- $d =$ daily demand rate
- $t =$ length of production run in days, or period over which the order will arrive

Maximum inventory level

\[
\text{Total produced} = Q = pt
\]

since

\[
\text{Total produced} = Q = pt
\]

we know

\[
t = \frac{Q}{p}
\]

Maximum inventory level

\[
= pt - dt = p \frac{Q}{p} - d \frac{Q}{p} = Q \left(1 - \frac{d}{p}\right)
\]
Annual Carrying Cost for Production Run Model

- Since the average inventory is one-half the maximum

\[
\text{Average inventory} = \frac{Q}{2} \left(1 - \frac{d}{p}\right)
\]

therefore

\[
\text{Annual holding cost} = \frac{Q}{2} \left(1 - \frac{d}{p}\right) C_h
\]

Annual Setup Cost for Production Run Model

- Setup cost replaces ordering cost when a product is produced over time

\[
\text{Number of production runs (or orders)} = \frac{D}{Q}
\]

\[
\text{Annual setup cost} = \frac{D}{Q} C_s
\]

and

\[
\text{Annual ordering cost} = \frac{D}{Q} C_o
\]

Determining the Optimal Production Quantity

- By setting setup costs equal to holding costs, we can solve for the optimal order quantity

\[
\frac{Q}{2} \left(1 - \frac{d}{p}\right) C_h = \frac{D}{Q} C_s
\]

Solving for Q, we get

\[
Q^* = \sqrt{\frac{2DC_s}{C_h \left(1 - \frac{d}{p}\right)}}
\]

Production Run Model

- Summary of equations

\[
\text{Annual holding cost} = \frac{Q}{2} \left(1 - \frac{d}{p}\right) C_h
\]

\[
\text{Annual setup cost} = \frac{D}{Q} C_s
\]

Optimal production quantity \( Q^* \) is

\[
Q^* = \sqrt{\frac{2DC_s}{C_h \left(1 - \frac{d}{p}\right)}}
\]

- If the situation does not involve production but receipt of inventory over a period of time, use the same model but replace \( C_s \) with \( C_o \).
Brown Manufacturing Example

Brown Manufacturing produces commercial refrigeration units in batches

Annual demand = \( D = 10,000 \) units
Setup cost = \( C_s = \$100 \)
Carrying cost = \( C_h = \$0.50 \) per unit per year
Daily production rate = \( p = 80 \) units daily
Daily demand rate = \( d = 60 \) units daily

1) How many refrigeration units should Brown produce in each batch?
2) How long should the production cycle last?

1. \( Q^* = \frac{2DC_s}{C_h \left( 1 - \frac{d}{p} \right)} \)
2. \( Q^* = \frac{2 \times 10,000 \times 100}{0.5 \left( 1 - \frac{60}{80} \right)} \)
3. Production cycle = \( \frac{Q}{p} \)

-  \( = \frac{4,000}{80} = 50 \) days
-  \( = 2,000,000 \) units
-  \( = \sqrt{16,000,000} = 4,000 \) units

Brown Manufacturing Example

Program 6.2A

Program 6.2B
West Valve Example

West Valve sells industrial valves. It has an annual demand of 4000 units. The cost of each valve is $90, and the inventory carrying cost is estimated to be 10% of the cost of each valve. Barbara has made a study of the costs involved in placing an order for any of the valves that West Valve stocks, and she has concluded that the average ordering cost is $25 per order. Furthermore, it takes about two weeks for an order to arrive from the supplier, and during this time the demand per week is approximately 80. The order arrives in one batch.

1. What is the EOQ?
2. What is the ROP?
3. What is the average inventory? What is the annual holding cost?
4. How many orders per year would be placed? What is the annual ordering cost?

Quantity Discount Models

If quantity discounts are available, the basic EOQ model needs to be adjusted by adding the purchase or material cost because it becomes a relevant cost.

Total cost = Material cost + Ordering cost + Holding cost

\[ \text{Total cost} = DC + \frac{D}{Q} C_o + \frac{Q}{2} C_h \]

where

\begin{align*}
D &= \text{annual demand in units} \\
C_o &= \text{ordering cost of each order} \\
C &= \text{cost per unit} \\
C_h &= \text{holding or carrying cost per unit per year}
\end{align*}

Since holding cost is based on the cost of the item, it is convenient to express it as

\[ I = \text{holding cost as a percentage of the unit cost} \ (C) \]

Total cost = \( DC + \frac{D}{Q} C_o + \frac{Q}{2} C_h \)

where

\begin{align*}
D &= \text{annual demand in units} \\
C_o &= \text{ordering cost of each order} \\
C &= \text{cost per unit} \\
C_h &= \text{holding or carrying cost per unit per year}
\end{align*}
### Quantity Discount Models

- A typical quantity discount schedule

<table>
<thead>
<tr>
<th>DISCOUNT NUMBER</th>
<th>DISCOUNT QUANTITY</th>
<th>DISCOUNT (%)</th>
<th>DISCOUNT COST ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 to 999</td>
<td>0</td>
<td>5.00</td>
</tr>
<tr>
<td>2</td>
<td>1,000 to 1,999</td>
<td>4</td>
<td>4.80</td>
</tr>
<tr>
<td>3</td>
<td>2,000 and over</td>
<td>5</td>
<td>4.75</td>
</tr>
</tbody>
</table>

Table 6.3

- Buying at the lowest unit cost is not always the best choice – the carrying cost goes up as the discount quantity increases

---

### Brass Department Store Example

- Brass Department Store stocks toy race cars
- Their supplier has given them the quantity discount schedule shown in Table 6.3
  - Annual demand is 5,000 cars, ordering cost is $49, and holding cost is 20% of the cost of the car
- The first step is to compute EOQ values for each discount

$$\text{EOQ}_1 = \sqrt{\frac{2 \times 5000 \times 49}{0.2 \times 5.00}} = 700 \text{ cars per order}$$

$$\text{EOQ}_2 = \sqrt{\frac{2 \times 5000 \times 49}{0.2 \times 4.80}} = 714 \text{ cars per order}$$

$$\text{EOQ}_3 = \sqrt{\frac{2 \times 5000 \times 49}{0.2 \times 4.75}} = 718 \text{ cars per order}$$

---

### Brass Department Store Example

- The second step is adjust quantities below the allowable discount range
- The EOQ for discount 1 is allowable
- The EOQs for discounts 2 and 3 are outside the allowable range and have to be adjusted to the smallest quantity possible to purchase and receive the discount

$$Q_1 = 700$$

$$Q_2 = 1,000$$

$$Q_3 = 2,000$$
The third step is to compute the total cost for each discount price:

\[
\text{Total Cost} = DC + \left(\frac{D}{Q}\right)C_o + \left(\frac{Q}{2}\right)IC
\]

- \(D = 5000\) cars; \(C_o = $49\); \(I = 20\%\)

<table>
<thead>
<tr>
<th>Discount Number</th>
<th>Unit Price (C)</th>
<th>Order Quantity (Q)</th>
<th>Annual Material Cost ($)</th>
<th>Annual Ordering Cost ($)</th>
<th>Annual Carrying Cost ($)</th>
<th>Total ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5.00</td>
<td>700</td>
<td>25,000</td>
<td>350.00</td>
<td>350.00</td>
<td>25,700.00</td>
</tr>
<tr>
<td>2</td>
<td>4.80</td>
<td>1,000</td>
<td>24,000</td>
<td>480.00</td>
<td>480.00</td>
<td>24,725.00</td>
</tr>
<tr>
<td>3</td>
<td>4.75</td>
<td>2,000</td>
<td>23,750</td>
<td>950.00</td>
<td>950.00</td>
<td>24,822.50</td>
</tr>
</tbody>
</table>

Table 6.4

The fourth step is to choose the alternative with the lowest total cost.

If EOQ assumptions are met, the stockouts can be completely avoided. However, if demand or the lead time are uncertain, the exact demand during lead time (ROP) will not be known with certainty. To prevent stockouts, it is necessary to carry extra inventory called safety stock. Safety stock can prevent stockouts when demand is unusually high. Safety stock can be implemented by adjusting the ROP.
The basic ROP equation is

$$\text{ROP} = d \times L$$

- $d$ = daily demand (or average daily demand)
- $L$ = order lead time or the number of working days it takes to deliver an order (or average lead time)

A safety stock variable is added to the equation to accommodate uncertain demand during lead time

$$\text{Adjusted ROP} = d \times L + SS$$

where

$SS$ = safety stock

When EOQ is fixed and the ROP is used to place orders, stockouts can only occur during the lead time

The objective is to find the safety stock quantity that will minimize the total of stockout cost and additional holding cost for the safety stock

To compute the total cost, it is necessary to know the stockout cost per unit and the probability distribution of demand during lead time

Estimating stockout costs can be difficult as there are direct and indirect costs

- Current lost sales – direct
- Future lost sales due to loss of goodwill – indirect
ABCO Example with Known Stockout Costs

- ABCO, Inc. has determined its ROP is 50 units
- The carrying cost per unit per year is $5
- The stockout cost is $40 per unit
- The probability distribution of demand during lead time is shown below
- The optimal number of orders per year is 6

<table>
<thead>
<tr>
<th>NUMBER OF UNITS</th>
<th>PROBABILITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.2</td>
</tr>
<tr>
<td>40</td>
<td>0.2</td>
</tr>
<tr>
<td>50</td>
<td>0.3</td>
</tr>
<tr>
<td>60</td>
<td>0.2</td>
</tr>
<tr>
<td>70</td>
<td>0.1</td>
</tr>
<tr>
<td>ROP</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 6.5

Determining the safety stock (SS) level that will minimize total expected cost is a decision making under risk problem (ROP = 50)

<table>
<thead>
<tr>
<th>PROBABILITY</th>
<th>0.20</th>
<th>0.20</th>
<th>0.30</th>
<th>0.20</th>
<th>0.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALTERNATIVE</td>
<td>Safety</td>
<td>0</td>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>Demand</td>
<td></td>
<td>$0</td>
<td>$0</td>
<td>$0</td>
<td>$2,400</td>
</tr>
<tr>
<td>Stock</td>
<td></td>
<td>$50</td>
<td>$50</td>
<td>$50</td>
<td>$50</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>30</td>
<td></td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 6.6

Note: The calculation here is different from that in the book.

ABCO Example with Known Stockout Costs

Stockout cost = # of units short × Stockout cost per unit × # of orders per year

Additional carrying cost = # of surplus units × Carrying cost per unit per year

When the SS is equal to 0, there is no additional carrying cost. Total cost is:
- = 0 (if inventory level ≥ demand over lead time)
- = stockout cost (if inventory level < demand over lead time)

When the SS is greater than 0, total cost is:
- = additional carrying cost (if inventory ≥ demand over lead time)
- = additional carrying cost + stockout cost (if inventory level < the demand over the lead time)

The minimum EMV of the total cost is $100

Figure 6.8
**ABCO Example with Known Stockout Costs**

- QM programs calculate the total costs differently from that in the book.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ABCO Example</td>
<td>Inventory</td>
<td>Safety stock - marginal analysis</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td>2</td>
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</tr>
<tr>
<td>7</td>
<td>Data</td>
<td>Reorder point</td>
<td>50</td>
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<td></td>
<td></td>
<td></td>
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<td>8</td>
<td></td>
<td>Annual carrying cost</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>9</td>
<td></td>
<td>Stockout cost</td>
<td>40</td>
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<td></td>
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<tr>
<td>10</td>
<td></td>
<td>Orders per year</td>
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<td></td>
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<td></td>
<td></td>
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<td>11</td>
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<td></td>
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<tr>
<td>12</td>
<td>Profit Table</td>
<td>Probability</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>Safety stock/Demand</td>
<td>10</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td></td>
<td></td>
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<td>14</td>
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<td>18</td>
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<td></td>
</tr>
</tbody>
</table>

**Safety Stock with Unknown Stockout Costs**

- There are many situations when stockout costs are unknown or extremely difficult to determine.
  - E.g. parts used for maintenance under warranty, lifesaving drugs, etc.
- An alternative approach to determining safety stock levels is to use a **service level**.
- A service level is the percent of time you will not be out of stock of a particular item.

  \[
  \text{Service level} = 1 - \text{Probability of a stockout}
  \]

  or

  \[
  \text{Probability of a stockout} = 1 - \text{Service level}
  \]

**Normal Distribution**

- Inventory demand is normally distributed during the reorder period.
- The normal distribution is symmetrical, with the midpoint \((\mu)\) representing the mean demand (ROP).
- The area below the curve represents the probability of the demand.
- The standard deviation specifies a range of demand \((\mu \pm 1\sigma)\) that counts for 68% of all possible demands.

**Normal Distribution Examples**

- Let \(\mu = 300, \sigma = 50\)
  - \(\text{P(demand}>300) = ?\) (prob. of stockout when \(SS=0\))
  - \(\text{P}(250<\text{demand}<350) = ?\)
  - \(\text{P(demand}>350) = ?\) (prob. of stockout when \(SS=50\))
Hinsdale Company Example

- Inventory demand is normally distributed during the reorder period
- Mean is 350 units and standard deviation is 10 units
- They want stockouts to occur only 5% of the time (service level = 95%)

\[ \mu = \text{Mean demand (ROP) = 350} \]
\[ \sigma = \text{Standard deviation = 10} \]
\[ X = \text{Mean demand + Safety stock} \]
\[ SS = \text{Safety stock} = X - \mu = Z\sigma \]
\[ Z = \frac{X - \mu}{\sigma} = \frac{SS}{\sigma} \]

\[ \mu = \text{Mean demand} = 350 \]
\[ \sigma = \text{Standard deviation} = 10 \]
\[ X = \text{Mean demand} + \text{Safety stock} \]
\[ SS = \text{Safety stock} = X - \mu = Z\sigma \]
\[ Z = \frac{X - \mu}{\sigma} = \frac{SS}{\sigma} \]

\[ SS = 1.65(10) = 16.5 \text{ units, or 17 units} \]

\[ \mu = \text{Mean demand} = 350 \]
\[ SS = \text{Safety stock} = X - \mu = Z\sigma \]
\[ SS = 1.65(10) = 16.5 \text{ units, or 17 units} \]

Cost of different service levels (assuming the carrying cost \(C_h = \$1\) per unit per year)

<table>
<thead>
<tr>
<th>SERVICE LEVEL (%)</th>
<th>Z VALUE FROM NORMAL CURVE TABLE</th>
<th>SAFETY STOCK (UNITS)</th>
<th>CARRYING COST ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>1.28</td>
<td>12.8</td>
<td>12.80</td>
</tr>
<tr>
<td>91</td>
<td>1.34</td>
<td>13.4</td>
<td>13.40</td>
</tr>
<tr>
<td>92</td>
<td>1.41</td>
<td>14.1</td>
<td>14.10</td>
</tr>
<tr>
<td>93</td>
<td>1.48</td>
<td>14.8</td>
<td>14.80</td>
</tr>
<tr>
<td>94</td>
<td>1.55</td>
<td>15.5</td>
<td>15.50</td>
</tr>
<tr>
<td>95</td>
<td>1.65</td>
<td>16.5</td>
<td>16.50</td>
</tr>
<tr>
<td>96</td>
<td>1.75</td>
<td>17.5</td>
<td>17.50</td>
</tr>
<tr>
<td>97</td>
<td>1.88</td>
<td>18.8</td>
<td>18.80</td>
</tr>
<tr>
<td>98</td>
<td>2.05</td>
<td>20.5</td>
<td>20.50</td>
</tr>
<tr>
<td>99</td>
<td>2.33</td>
<td>23.3</td>
<td>23.30</td>
</tr>
<tr>
<td>99.99</td>
<td>3.72</td>
<td>37.2</td>
<td>37.20</td>
</tr>
</tbody>
</table>

Table 6.7
Hinsdale Company Example

- Cost gets extremely large when service level is greater than 98%
- This was developed for a specific case, but the general shape of the curve is the same for all service-level problems

![Figure 6.8](image)

Service Level (%) vs. Inventory Carrying Costs ($)

Single-Period Inventory Models

- So far we have been assuming that there is a constant demand for the products, i.e. ordered products will be sold in the future
- However, some products have no future value beyond the current period (weekly magazines, daily newspapers, certain food with a short life, some seasonal clothes...)
- These situations are called news vendor problems or single-period inventory models
- Though payoff tables can be used, the large number of alternatives and states of nature involved make them too difficult to use
- Marginal analysis is a simpler approach to find the optimal inventory level for news vendor problems

Single-Period Inventory Models

- With a manageable number of states of nature and alternatives where the probability of each state of nature is known, marginal analysis with discrete distributions can be used
- When there are a large number of alternatives or states of nature and the probability distribution can be described with a normal distribution, marginal analysis with the normal distribution may be used
- Marginal analysis is based on manipulating marginal profit (MP) and marginal loss (ML)
  - MP is the additional profit achieved if one additional unit is stocked and sold
  - ML is the loss that occurs when an additional unit is stocked but cannot be sold
  - Decision rule – we stock an additional unit only if the expected marginal profit for that unit exceeds the expected marginal loss
  - Assume
    \[
    P = \text{probability that demand will be greater than or equal to a given supply (or the probability of selling at least one additional unit)}
    \]
    \[
    1 - P = \text{probability that demand will be less than supply (or the probability that one additional unit will not sell)}
    \]
Marginal Analysis with Discrete Distributions

**Steps of Marginal Analysis with Discrete Distributions**

1. Determine the value of \( \frac{ML}{ML + MP} \) for the problem
2. Construct a probability table and add a cumulative probability column
3. Keep ordering inventory as long as the probability \( P \) of selling at least one additional unit is greater than \( \frac{ML}{ML + MP} \)

**Café du Donut Example**

- The café buys donuts each day for $4 per carton of 2 dozen donuts
- Any cartons not sold are thrown away at the end of the day, for they are not fresh enough to meet the café’s standards
- If a carton is sold, the total revenue is $6
- The marginal profit per carton is
  \[ MP = \text{Marginal profit} = \$6 - \$4 = \$2 \]
- The marginal loss is $4 per carton since cartons cannot be returned or salvaged
### Café du Donut Example

#### Probability distribution of daily demand

<table>
<thead>
<tr>
<th>DAILY SALES (CARTONS OF DOUGHNUTS)</th>
<th>PROBABILITY ($P$) THAT DEMAND WILL BE AT THIS LEVEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.05</td>
</tr>
<tr>
<td>5</td>
<td>0.15</td>
</tr>
<tr>
<td>6</td>
<td>0.15</td>
</tr>
<tr>
<td>7</td>
<td>0.20</td>
</tr>
<tr>
<td>8</td>
<td>0.25</td>
</tr>
<tr>
<td>9</td>
<td>0.10</td>
</tr>
<tr>
<td>10</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1.00</strong></td>
</tr>
</tbody>
</table>

Table 6.8

### Step 1. Determine the value of $\frac{ML}{ML + MP}$ for the decision rule

\[
P \geq \frac{ML}{ML + MP} = \frac{4}{4 + 2} = \frac{4}{6} = 0.67
\]

\[
P \geq 0.67
\]

#### Step 2. Add a new cumulative probability column to the table to reflect the probability that doughnut sales will be at each level or greater

<table>
<thead>
<tr>
<th>DAILY SALES (CARTONS OF DOUGHNUTS)</th>
<th>PROBABILITY ($P$) THAT DEMAND WILL BE AT THIS LEVEL</th>
<th>PROBABILITY ($P$) THAT DEMAND WILL BE AT THIS LEVEL OR GREATER</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.05</td>
<td>1.00 ≥ 0.67</td>
</tr>
<tr>
<td>5</td>
<td>0.15</td>
<td>0.95 ≥ 0.67</td>
</tr>
<tr>
<td>6</td>
<td>0.15</td>
<td>0.80 ≥ 0.67</td>
</tr>
<tr>
<td>7</td>
<td>0.20</td>
<td>0.65</td>
</tr>
<tr>
<td>8</td>
<td>0.25</td>
<td>0.45</td>
</tr>
<tr>
<td>9</td>
<td>0.10</td>
<td>0.20</td>
</tr>
<tr>
<td>10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1.00</strong></td>
<td><strong>1.00</strong></td>
</tr>
</tbody>
</table>

Table 6.9

#### Step 3. Keep ordering additional cartons as long as the probability of selling at least one additional carton is greater than or equal to 0.67, which is the indifference probability

\[
P \text{ at 6 cartons} = 0.80 \geq 0.67
\]

Therefore Café du Donut should buy 6 cartons of donuts each day, since 0.8 is the smallest probability value that is greater than or equal to 0.67.
Marginal Analysis with the Normal Distribution

- Product demand or sales usually follow a normal distribution
- We first need to find four values
  1. The average or mean sales for the product, \( \mu \)
  2. The standard deviation of sales, \( \sigma \)
  3. The marginal profit for the product, \( MP \)
  4. The marginal loss for the product, \( ML \)
- We let \( X^* \) = optimal stocking level

Steps of Marginal Analysis with the Normal Distribution

1. Determine the value of \( \frac{ML}{ML + MP} \) for the problem
2. Use the above value to locate \( P \) on the normal distribution (Appendix A) and find the associated \( Z \)-value
3. Find \( X^* \) using the relationship
   \[
   Z = \frac{X^* - \mu}{\sigma}
   \]
   to solve for the resulting stocking policy:
   \[
   X^* = \mu + Z \sigma
   \]

Newspaper Example

- Demand for the *Chicago Tribune* at Joe’s Newsstand averages 60 papers a day with a standard deviation of 10
- The marginal loss is 20 cents and the marginal profit is 30 cents
- Step 1. Joe should stock the Tribune as long as the probability of selling the last unit is at least \( ML/(ML + MP) \):
  \[
  \frac{ML}{ML + MP} = \frac{20 \text{ cents}}{20 \text{ cents} + 30 \text{ cents}} = \frac{20}{50} = 0.40
  \]
  Let \( P = 0.40 \)

Step 2. Using the normal distribution in Figure 6.11, we find the appropriate \( Z \) value

\[
Z = 0.25 \text{ standard deviations from the mean}
\]

Area under the curve is \( 1 - 0.40 = 0.60 \)

\( Z = 0.25 \)

Mean Daily Sales

Area under the curve is 0.40

\( \mu = 60 \)

\( X^* \)

\( X \) = Demand

Figure 6.11 Optimal Stocking Policy (62 Newspapers)
Newspaper Example

Step 3. In this problem, \( \mu = 60 \) and \( \sigma = 10 \), so

\[
0.25 = \frac{X^* - 60}{10}
\]

or

\[
X^* = 60 + 0.25(10) = 62.5, \text{ or } 62 \text{ newspapers}
\]

Joe should order 62 newspapers since the probability of selling 63 newspapers is slightly less than 0.40.

Newspaper Example

- When \( P > 0.50 \), there is a change in finding \( Z \) value
- Joe also stocks the *Chicago Sun-Times*
- Marginal loss and profit are 40 cents and 10 cents
- Daily sales average 100 copies with a standard deviation of 10 papers

\[
\frac{\text{ML}}{\text{ML} + \text{MP}} = \frac{40 \text{ cents}}{40 \text{ cents} + 10 \text{ cents}} = \frac{40}{50} = 0.80
\]

Since the curve for \( P = 0.2 \) is symmetric to the curve for \( P = 0.8 \), we look for the \( Z \) value for \( P = 0.8 \) in Appendix A, and then multiply the \( Z \) value by \(-1\)

\[
Z = -0.85 \text{ standard deviations from the mean}
\]

ABC Analysis

- In real situation the inventory usually consists of many different types of items
- The purpose of ABC analysis is to divide the inventory into three groups based on the overall inventory value of the items and focusing on managing the most valuable group of inventory
ABC Analysis

- Group A items account for the major portion of inventory costs
  - Typically about 80% of the dollar value but only 20% of the quantity of items
  - Great care should be taken in forecasting the demand and develop optimal inventory management policy
- Group B items are more moderately priced
  - May represent 15% of the cost and 30% of the quantity
  - Quantitative analysis and control may or may not be necessary
- Group C items are very low cost but high volume
  - Constitute 5% of the cost but 50% of the quantity
  - It is not cost effective to spend a lot of time managing these items

Summary of ABC analysis

<table>
<thead>
<tr>
<th>INVENTORY GROUP</th>
<th>DOLLAR USAGE (%)</th>
<th>INVENTORY ITEMS (%)</th>
<th>ARE QUANTITATIVE CONTROL TECHNIQUES USED?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>80</td>
<td>20</td>
<td>Yes</td>
</tr>
<tr>
<td>B</td>
<td>15</td>
<td>30</td>
<td>In some cases</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>50</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 6.10

A table of six items in an inventory

<table>
<thead>
<tr>
<th>ID CODE</th>
<th>UNIT COST ($)</th>
<th># OF ITEMS</th>
<th>TOTAL COST ($)</th>
<th>% OF TOTAL ITEMS</th>
<th>% OF TOTAL COST</th>
<th>INVENTORY GROUP</th>
</tr>
</thead>
<tbody>
<tr>
<td>XX1</td>
<td>5.84</td>
<td>1200</td>
<td>7,008</td>
<td>18.8</td>
<td>7</td>
<td>B</td>
</tr>
<tr>
<td>B66</td>
<td>5.40</td>
<td>1110</td>
<td>5,994</td>
<td>17.4</td>
<td>6</td>
<td>B</td>
</tr>
<tr>
<td>3CPO</td>
<td>1.12</td>
<td>896</td>
<td>1,003.52</td>
<td>14.0</td>
<td>1</td>
<td>C</td>
</tr>
<tr>
<td>33CP</td>
<td>74.54</td>
<td>1104</td>
<td>82,292.16</td>
<td>17.3</td>
<td>82</td>
<td>A</td>
</tr>
<tr>
<td>R2D2</td>
<td>2.00</td>
<td>1110</td>
<td>2,220</td>
<td>17.4</td>
<td>2</td>
<td>C</td>
</tr>
<tr>
<td>RMS</td>
<td>2.08</td>
<td>961</td>
<td>1,998.88</td>
<td>15.1</td>
<td>2</td>
<td>C</td>
</tr>
</tbody>
</table>

TOTAL 6381 100,516.56 100 100

Which item needs strict control?

Just-in-Time Inventory Control

- To achieve greater efficiency in the production process, organizations have tried to have less in-process inventory on hand – this is known as JIT inventory
- The inventory arrives just in time to be used during the manufacturing process
  - Producing the necessary items in necessary quantities at the necessary time
  - A philosophy of continuous improvement in which non-value-adding activities (or wastes) are identified and removed
- One technique of implementing JIT is a manual procedure called kanban
Kanban in Japanese means “card” and used as a signal
With a dual-card kanban system, there is a conveyance kanban, or C-kanban, and a production kanban, or P-kanban
Kanban systems are quite simple, but they require considerable discipline
As there is little inventory to cover variability, the schedule must be followed exactly

1. A user takes a container of parts or inventory along with its C-kanban to his or her work area
When there are no more parts or the container is empty, the user returns the container along with the C-kanban to the storage area
2. At the storage area, there is a full container of parts along with a P-kanban. The user detaches the P-kanban from the full container and takes the container and the C-kanban back to his or her area for immediate use
3. The detached P-kanban goes back to the producer area along with the empty container
The P-kanban is a signal that new parts are to be manufactured or that new parts are to be placed in the container
4. When the container is filled, the P-kanban is attached to the container and the container goes back to the storage area
5. This process repeats itself during the typical workday

The Kanban System

Figure 6.17
The Kanban System

- Inventory arrives at the user area just when it is needed
- Parts are produced only when they are required in the user area
- The Kanban system lowers inventory levels, reduces costs, and makes a manufacturing process more efficient

Homework Assignment

http://www.sci.brooklyn.cuny.edu/~dzhu/busn3430/