Follow these instructions carefully:

Work on the paper provided; do not use your own paper. Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet). On the top of each page, print your name (encircle your last name) and indicate the number of the problem you are working on by writing e.g. “Problem #4”. Always encircle your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing “Answer to Part b)” (the number of the problem should be on the top of the page). Do not use a red pen or a red pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the same amount of credit.

1. Calculate the following limits:
   a) \( \lim_{x \to 0} \frac{(1 - 2x)^{1/x}}{x} \),
   b) \( \lim_{x \to 1} \left( \frac{1}{\ln x} - \frac{1}{x - 1} \right) \).

2. a) Write the integral that finds the length of the curve \( r = 3 - 2 \sin \theta \) in polar coordinates. Do not calculate the integral.
   b) Write the integral that finds the area inside the curve \( r = 4 \sin \theta \) and outside the curve \( r = 3 - 2 \sin \theta \).
   Do not calculate the integral.

3. Decide whether the following improper integrals are convergent or divergent. Give clear reasons for your answer (no credit will be given for a correct answer unless the correct reason is also given). Do not calculate the integrals.
   a) \( \int_2^{+\infty} \frac{dx}{x^2 \ln x} \),
   b) \( \int_0^{+\infty} \frac{x \, dx}{x^2 + 1} \),
   c) \( \int_1^{+\infty} \frac{\ln x \, dx}{x^2 + 1} \).

4. Decide whether or not each of the following sequences is convergent. Give reasons for your answers. If the given sequence is convergent, find its limit.
   a) \( a_n = \frac{n - 1}{n + 1} \),
   b) \( a_n = \frac{2^n - 1}{2^n + 1} \),
   c) \( a_n = \cos n\pi \),
   d) \( a_n = \frac{\cos n\pi}{\sqrt{n}} \),
   e) \( a_n = \frac{\cos n}{n^2} \),
   f) \( a_n = (-1)^n \frac{n}{n + 1} \).

5. a) Decide whether the sum \( \sum_{n=3}^{\infty} \frac{1}{n^2 - 1} \) is convergent. If it is convergent, find its limits.
   b) Decide whether the series \( \sum_{n=2}^{\infty} \frac{2^{3n-5}}{3^{2n-3}} \) is convergent. If it is convergent, find its sum.