Follow these instructions carefully:

Work on the paper provided; do not use your own paper. Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet). On the top of each page, print your name (encircle your last name) and indicate the number of the problem you are working on by writing e.g. “Problem #4”. Always encircle your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing “Answer to Part b)” (the number of the problem should be on the top of the page). Do not use a red pen or a red pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the same amount of credit.

1. a) Decide whether each of the following is true or false. Do not explain.
   (i) \{1,3,5\} = \{3,1,1,5,3\};
   (ii) \emptyset \in \{\emptyset\};
   (iii) \emptyset \subseteq \{3,4\};
   (iv) \{2\} \in \{\{2\}\};
   (v) \{2\} \subseteq \{\{2\}\};
   (vi) for any sets \(A\) and \(B\), \(A \cup B \subseteq A \cap B\);
   (vii) for any sets \(A\) and \(B\), \(A \cap B \subseteq A \cup B\);
   (viii) for any set \(A\), \(A \cap \emptyset = A\);
   (ix) for any set \(A\), \(A \cup \emptyset = A\).
   b) For any real number \(x\), let \(A_x\) be the interval \((-\infty, x)\). Describe the sets (i) \(\bigcup_{x \in (-\infty, +\infty)} A_x\), and (ii) \(\bigcap_{x \in (-\infty, +\infty)} A_x\).

2. a) Write the truth table for the formula \((P \land \sim Q) \implies R\).
   b) Let \(x\) and \(y\) run over integers. Decide whether each of the following is true or false:
   i) \((\forall x)(\exists y)[x \neq y]\).
   ii) \((\exists y)(\forall x)[x \neq y]\).
   Explain.
   c) Move the negation all the way inside in the formula
      \(\sim (\forall x)(\exists y)(\forall z) [P(x) \land (Q(y) \implies \sim R(z))]\).

3. Decide whether each of the following logical equivalences is true or false. In each case, explain why the equivalence in question is true, or why it is false. If the assertion is false, also give an example for \(P(x)\) and \(Q(x)\) that makes the assertion false.

   **Note:** There is a subtle difference between logical equivalence \(\equiv\) and the biconditional \(\iff\). Logical equivalence \(\equiv\) means that the formulas on the two sides have exactly the same meaning, while the biconditional \(\iff\) means that the formulas in a specific realization – when \(x\) ranges over integers, say, and \(P(x)\) and \(Q(x)\) are chosen in a specific way – the two sides are true at the same time.

   a) \((\exists x)\left(P(x) \lor Q(x)\right) \equiv \left(\exists x\right)P(x) \lor \left(\exists x\right)Q(x)\),
   b) \((\exists x)\left(P(x) \implies Q(x)\right) \equiv \left(\forall x\right)P(x) \implies \left(\exists x\right)Q(x)\),
   c) \((\exists x)\left(P(x) \implies Q(x)\right) \equiv \left(\exists x\right)P(x) \implies \left(\exists x\right)Q(x)\),

4. a) Define what it means for an integer to be (i) even, and (ii) odd. (Give two separate definitions, one for (i) and one for (ii).)
   b) Prove the following: If \(n\) is an odd integer, then \(n^2 - 4n + 5\) is even.
   c) Prove the following: For every integer \(n\), if \(5n - 3\) is even, then \(n\) is odd.

5. Prove the following: If \(n\) is an integer such that \(5n^2 - 2n\) is even, then \(3n - 1\) is odd.