Follow these instructions carefully:

Work on the paper provided; do not use your own paper. *Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet).* On the top of each page, print your name (encircle your last name) and indicate the number of the problem you are working on by writing e.g. “Problem #4”. Always encircle your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing “Answer to Part b)” (the number of the problem should be on the top of the page). Do not use a red pen or a red pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the same amount of credit.

1. a) Disprove the statement that if \(a\) and \(b\) are positive real numbers then \(\sqrt{a + b} = \sqrt{a} + \sqrt{b}\).
   b) Prove that the product of a nonzero rational number and of an irrational number is irrational.

2. a) Let \(p\) be a prime number and let \(a\) and \(b\) be integers. Prove that if \(p \mid ab\) then \(p \mid a\) or \(p \mid b\).
   b) Use the result in Part a) to show the following. Let \(a, b, c\) be integers, and let \(p\) be a prime. Assume that \(c \not\equiv 0 \pmod{p}\) and \(ac \equiv bc \pmod{p}\). Then \(a \equiv b \pmod{p}\).

3. a) Let \(R\) be a relation. Define the domain and the range of \(R\).
   b) Prove that \(\sqrt{7}\) is irrational.

4. a) Define what an equivalence relation is.
   b) Let \(R\) be the relation on the set of integers defined by \(aRb\) for integers \(a\) and \(b\) if and only if \(a \mid b\) or \(b \mid a\). Prove or disprove that \(R\) is an equivalence relation.
   c) Let \(A\) be a set and let \(R_1\) and \(R_2\) be equivalence relations on \(A\). Prove that \(R = R_1 \cap R_2\) is also an equivalence relation.

5. a) Let \(A, B, C\) be sets, and let \(f : A \to B\) and \(g : B \to C\) be functions. Assume that \(f\) is onto \(B\) and \(g\) is onto \(C\). Prove that \(g \circ f\) is onto \(C\).
   b) Show that the function \(f : \mathbb{Z}_5 \to \mathbb{Z}_5\) defined as \(f([x]) = [4x + 2]\) for \(x \in \mathbb{Z}\) is one-to-one and onto \(\mathbb{Z}_5\).