Exam 1, Mathematics 2001, Section ETR6
6:30 pm–7:40 pm, September 26, 2013, IA-137
Instructor: Attila Máté

Follow these instructions carefully:

Work on the paper provided; do not use your own paper. Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet). On the top of each page, print your name (encircle your last name) and indicate the number of the problem you are working on by writing e.g. “Problem #4”. Always encircle your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing “Answer to Part b)” (the number of the problem should be on the top of the page). Do not use a red pen or a red pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the same amount of credit.

1. a) Decide which which each of the following is true or false. Do not explain.
   (i) \(\{1, 2, 3, 2, 1\} = \{1, 2, 3\}\),
   (ii) \(\emptyset \in \{1, 2, \emptyset\}\),
   (iii) \(\emptyset \subseteq \{1, 2, 3\}\),
   (iv) \(\emptyset \in \{\{\emptyset\}\}\),
   (v) \(\{\emptyset\} \subseteq \{\{\emptyset\}\}\),
   (vi) for any sets \(A\) and \(B\), \(A \subseteq A \cup B\),
   (vii) for any sets \(A\) and \(B\), \(A \subseteq A \cap B\),
   (viii) for any sets \(A\) and \(B\), \(A \cap B \subseteq A\).

   b) For any positive real number \(x\), let \(A_x\) be the interval \([0, x]\). Describe the sets (i) \(\bigcup_{x \in (0, 1)} A_x\), and (ii) \(\bigcap_{x \in (0, 1)} A_x\).

2. a) Write the truth table for the formula \(P \implies (Q \implies \sim R)\)
   b) Let \(x\) and \(y\) run over integers. Decide whether each of the following is true or false:
      (i) \((\forall x)(\exists y)[x > y]\).
      (ii) \((\exists y)(\forall x)[x > y]\).

      Explain.

   c) Move the negation all the way inside in the formula

\[\sim (\exists x)(\exists y)(\forall z)(P(x) \vee (Q(y) \implies R(z))].\]

3. Decide whether each of the following is true or false. In each case, explain in your own words why the formula in question is true, or why it is false. If the formula is false, also give an example for \(P(x)\) and \(Q(x)\) that makes the formula false.
   a) \((\forall x)(P(x) \& Q(x)) \iff (\forall x)P(x) \& (\forall x)Q(x))\),
   b) \((\exists x)(P(x) \& Q(x)) \iff (\exists x)P(x) \& (\exists x)Q(x))\),
   c) \((\forall x)(P(x) \vee Q(x)) \iff (\forall x)P(x) \vee (\forall x)Q(x))\),
   d) \((\exists x)(P(x) \vee Q(x)) \iff (\exists x)P(x) \vee (\exists x)Q(x))\).

4. a) Define what it means for an integer to be (i) even, and (ii) odd. (Give two separate definitions, one for (i) and one for (ii)).
   b) Prove the following: If \(n\) is an odd integer, \(3n^2 + 4n - 5\) is even.
   c) Prove the following: For every integer \(n\), if \(5n + 2\) is odd, then \(n\) is odd.

5. Prove the following: If \(n\) is an integer such that \(3n + 2\) is even, then \(5n + 7\) is odd.