Exam 1, Mathematics 4701, Section TY4
4:15 pm–5:50 pm, October 2, 2014, IA-236
Instructor: Attila Máté

Follow these instructions carefully:
Work on the paper provided; do not use your own paper. Work only on one problem on each sheet (you should not work on two different problems on the two sides of the same sheet). On the top of each page, print your name (encircle your last name) and indicate the number of the problem you are working on by writing e.g. “Problem #4”. Always encircle your final answer. If there are several parts to a problem, always indicate the part that you are answering, e.g. by writing “Answer to Part b)” (the number of the problem should be on the top of the page). Do not use a red pen or a red pencil. Do not write in the corner covered up by the staple (top left corner on the front side, top right corner on the back side). Each problem is worth the same amount of credit.

1. Calculate $x - \sqrt{x^2 - 2}$ for $x = 1,000,000$ with 6 significant digit accuracy. Avoid the loss of significant digits.

2. a) Evaluate $(2x - y^3)^2$

b) The leading term of the Newton interpolation polynomial $P(x)$ to a function $f$ with the nodes $x_1, x_2, \ldots, x_n$ is

$$f[x_1, x_2, \ldots, x_n]x^{n-1}.$$

Using this, show that

$$f[x_1, x_2, \ldots, x_n] = f[x_{i_1}, x_{i_2}, \ldots, x_{i_n}]$$

for any permutation $x_{i_1}, x_{i_2}, \ldots, x_{i_n}$ of the nodes $x_1, x_2, \ldots, x_n$. (All the nodes $x_1, x_2, \ldots, x_n$ are assumed to be distinct.)

3. Find the Newton-Hermite interpolation polynomial for $f(x)$ with $f(2) = 4$, $f'(2) = 15$, $f(4) = 10$, $f'(4) = 39$, $f''(4) = 28$.

a) First, write the divided difference table, using the points 2, 4 in natural order.

b) Using the divided difference table, write the Newton-Hermite interpolation polynomial using the order of points 2, 2, 4, 4, 4.

c) Using the divided difference table, write the Newton-Hermite interpolation polynomial using the order of points 4, 2, 4, 2, 4.

4. a) Consider the equation $f(x) = 0$ with $f(x) = 2 - x + \ln x$. Using Newton’s method with $x_0 = 3$ as a starting point, find the next approximation to the solution of the equation.

b) State the usual sufficient condition for the fixed-point iteration to converge when solving the equation $x = f(x)$.

c) The equation $e^x - x - 2 = 0$ has one positive solution and one negative solution. Rearrange the equation so that each of these solutions can be found by fixed-point iteration.

5. a) Evaluate the derivative of $P(x) = x^3 - 4x^2 + 6x + 4$ at $x = 2$ using Horner’s method. Show the details of your calculation.

b) Let $P$ and $Q$ be polynomials, let $x_0$ and $r$ be numbers, and assume that

$$P(x) = (x - x_0)Q(x) + r.$$

Show that $P'(x_0) = Q(x_0)$. 