CISC 3410 Fall 2012, Homework 5

1. Which, if any, of the following sentences is a proposition? In each case, justify your answer.
   (a) Twice two is four.
   (b) The square root of ten is three.
   (c) 5 > 6
   (d) Please write a specimen of your signature in the space provided.
   (e) Would you believe Lady Gaga once stood in exactly that spot?

2. Using the following propositions:
   
   \[
   p \quad \text{It's raining} \\
   q \quad \text{I have an umbrella} \\
   r \quad \text{I get wet}
   \]

   Formulate the following expressions in words:
   (a) \((p \land q)\)
   (b) \((p \land \neg q) \land r\)
   (c) \(\neg p \land \neg r\)
   (d) \(p \land (q \lor r)\)
   (e) \(\neg p \lor r\)

3. For each of the propositions in the previous question, write out the truth table.

4. For each of the propositions in question 2, say whether it is: consistent, inconsistent and/or a tautology.

5. Use the truth table method to decide whether the following statements are true:
   (a) \((p \land q) \models \neg(p \lor q)\)
   (b) \(p, (p \leftrightarrow q) \models p \land q\)
   (c) \((p \Rightarrow q) \models ((p \Rightarrow r) \Rightarrow (p \Rightarrow r))\)

6. There is a list of natural deduction proof rules for propositional logic on the class web page. Using these, try to prove the following:
   (a) \((p, p \Rightarrow (q \land r)) \vdash (p \land r)\)
   (b) \((p, p \Rightarrow (q \land r)) \vdash (s \lor r)\)
   (c) \((p \land (p \Rightarrow (q \land r))) \vdash (p \Rightarrow r)\)
   (d) \((p \Rightarrow (q \land r)) \vdash (p \Rightarrow r)\)
   (e) \((\neg q, p \leftrightarrow (q \land r)) \vdash \neg p\)