CISC 2210 – Introduction to Discrete Structures

Midterm 1 Exam

October 5, 2021

Id: .....

Problem	Maximum Points	Your Points
Sets 1	15	
Sets 2	15	
Sets 3	20	
Logic 1	20	
Logic 2	20	
Logic 3	10	
Total	100	

Structure, problem selection, and credit:

- You have 75 minutes to complete the exam.
- There are two parts: one for the topic of Sets and one for the topic of Logic. Each part contains three problems. See above the credit for each of the six problems for a total of 100 credits.
- You will only get partial credit if you fail to justify your answers. You will get 20% of the credit if you do not answer a problem. You will get zero credit for wrong answers.

Honor code: Students are expected to do this exam **by themselves** without any external help from other people, the Internet, books, or notes. Cheaters will be punished severely. At minimum, they will fail the exam, but they may fail the whole class. In addition, students who cheat risk disciplinary measures by Brooklyn College and CUNY.

1. **(15 credits)**

For two sets A and B, find a simpler form for the sets S and T.

$$S = (A \setminus B) \cup B \qquad T = A \cup (B \setminus A)$$

Justify your answers.

2. **(15 credits)**

In the Venn Diagrams for the three sets A, B, and C, mark the area representing the set

$$(A \cap \overline{B}) \cup (A \cap \overline{C})$$

Justify your answer.

3. **(20 credits)**

There are 100 students in the Computer Science program:

- 30 students can program in C, 40 students can program in Java, and 50 students can program in Python.
- Every student masters at least one language.
- However, if a student knows more than one language, then this student knows all three of the languages.

How many students can program in all three languages? Justify your answer.

Hint: A student either master one language or three languages. There are no other options.

4. **(20 credits)**

In the following truth table, the columns T_1, T_2, T_3, T_4, T_5 represent 5 different formulas. Below you can find 4 formulas in an arbitrary order. Match each formula with one of the T columns (one column does not represent any of the 4 formulas).

Justify your answers with few sentences.

x	y	z	w	T_1	T_2	T_3	T_4	T_5
T	T	T	T	F	T	F	T	T
T	T	T	F	F	T	T	T	F
T	T	F	T	F	T	T	T	T
T	T	F	F	T	T	F	T	F
T	F	T	T	T	T	T	T	T
T	F	T	F	T	F	F	T	F
T	F	F	T	T	F	F	F	T
T	F	F	F	F	F	T	F	F
F	T	T	T	T	T	T	T	T
F	T	T	F	T	F	F	F	T
F	T	F	T	T	F	F	T	T
F	T	F	F	F	F	T	F	T
F	F	T	T	T	T	F	T	T
F	F	T	F	T	F	T	F	T
F	F	F	T	T	F	T	F	F
F	F	F	F	F	F	F	F	F

Formula 1: $(x \wedge y) \vee (z \wedge w)$

Formula 2: $(x \vee w) \wedge (y \vee z)$

Formula 3: $(\neg x \wedge y) \vee (\neg x \wedge z) \vee (x \wedge w)$

Formula 4: $(x \wedge y) \oplus (z \vee w)$

5. **(20 credits)**

For $n \geq 1$, the goal is to count the number of truth assignments that satisfy the following formula on the $n+1$ variables $x, x_1, x_2, \ldots, x_n$:

$$x \oplus (x_1 \wedge x_2 \wedge x_3 \wedge \cdots \wedge x_n)$$

Recall that in total there are 2^{n+1} possible assignments.

Justify your answers to the following four parts.

(a) The case $n = 1$. How many truth assignments satisfy the formula: $x \oplus x_1$?

(b) The case $n = 2$. How many truth assignments satisfy the formula: $x \oplus (x_1 \wedge x_2)$?

(c) The case $n = 3$. How many truth assignments satisfy the formula: $x \oplus (x_1 \wedge x_2 \wedge x_3)$?

(d) The general case. How many truth assignments satisfy the formula: $x \oplus (x_1 \wedge x_2 \wedge x_3 \wedge \cdots \wedge x_n)$?

6. **(10 credits)**

In front of you there are three jars:

- One jar contains only Red marbles, one jar contains only Blue marbles, and one jar contains only Green marbles.
- One jar is labeled Red/Blue, one is labeled Blue/Green, and one is labeled Green/Red.
- A jar that is labeled with two colors must contain one of them, but it is unknown which one.

You are allowed to select one marble from any jar of your choice and observe its color. Then you must correct the labels to accurately indicate the color of the marbles in each jar.

Describe how to accomplish this task and explain why your method is always correct.