CISC 2210 – Introduction to Discrete Structures

Midterm 1 Exam

October 5, 2021

Id: .....

Problem	Maximum Points	Your Points
Sets 1	15	
Sets 2	15	
Sets 3	20	
Logic 1	20	
Logic 2	20	
Logic 3	10	
Total	100	

Structure, problem selection, and credit:

- You have 75 minutes to complete the exam.
- There are two parts: one for the topic of Sets and one for the topic of Logic. Each part contains three problems. See above the credit for each of the six problems for a total of 100 credits.
- You will only get partial credit if you fail to justify your answers. You will get 20% of the credit if you do not answer a problem. You will get zero credit for wrong answers.

Honor code: Students are expected to do this exam **by themselves** without any external help from other people, the Internet, books, or notes. Cheaters will be punished severely. At minimum, they will fail the exam, but they may fail the whole class. In addition, students who cheat risk disciplinary measures by Brooklyn College and CUNY.

1. **(15 credits)**

For two sets A and B, find a simpler form for the sets S and T.

$$S = A \cup (A \cap B) \qquad T = A \cap (A \cup B)$$

Justify your answers.

2. **(15 credits)**

In the Venn Diagrams for the three sets A, B, and C, mark the area representing the set

$$((A \cap B) \cup (A \cap C) \cup (B \cap C)) \setminus (A \cap B \cap C)$$

Justify your answer.

3. **(20 credits)**

After grading the Midterm exam, the professor created the following three lists of students:

- One list contained the names of all the 30 students whose grades were between 0 and 60.
- One list contained the names of all the 30 students whose grades were between 40 and 100.
- One list contained the names of all the 20 students whose grades were between 30 and 70.
- 15 students appeared in exactly two lists.

Answer the following two questions and justify your answers.

(a) How many students appeared in all three lists?

(b) How many students took the exam?

4. **(20 credits)**

In the following truth table, the columns T_1, T_2, T_3, T_4, T_5 represent 5 different formulas. Below you can find 4 formulas in an arbitrary order. Match each formula with one of the T columns (one column does not represent any of the 4 formulas).

Justify your answers with few sentences.

x	y	z	w	T_1	T_2	T_3	T_4	T_5
T	T	T	T	T	F	T	T	F
T	T	T	F	T	T	T	T	T
T	T	F	T	F	T	T	T	T
T	T	F	F	F	F	F	F	T
T	F	T	T	F	T	T	T	F
T	F	T	F	F	F	F	T	T
T	F	F	T	F	F	T	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T	F
F	T	T	F	F	F	T	T	T
F	T	F	T	T	F	F	T	T
F	T	F	F	F	T	F	F	T
F	F	T	T	T	F	F	F	T
F	F	T	F	F	T	F	F	F
F	F	F	T	T	T	F	F	F
F	F	F	F	F	F	F	F	F

Formula 1: $(x \vee y) \wedge (z \vee w)$

Formula 2: $(x \wedge w) \vee (y \wedge z)$

Formula 3: $(\neg x \vee y) \wedge (\neg x \vee z) \wedge (x \vee w)$

Formula 4: $(x \vee y) \oplus (z \wedge w)$

5. **(20 credits)**

For $n \geq 1$, the goal is to count the number of truth assignments that satisfy the following formula on the $n+1$ variables $y, y_1, y_2, \ldots, y_n$:

$$y \equiv (y_1 \vee y_2 \vee y_3 \vee \cdots \vee y_n)$$

Recall that in total there are 2^{n+1} possible assignments.

Justify your answers to the following four parts.

(a) The case $n = 1$. How many truth assignments satisfy the formula: $y \equiv y_1$?

(b) The case $n = 2$. How many truth assignments satisfy the formula: $y \equiv (y_1 \vee y_2)$?

(c) The case $n = 3$. How many truth assignments satisfy the formula: $y \equiv (y_1 \vee y_2 \vee y_3)$?

(d) The general case. How many truth assignments satisfy the formula: $y \equiv (y_1 \vee y_2 \vee y_3 \vee \cdots \vee y_n)$?

6. **(10 credits)**

In front of you, on a table, you see 3 cards: One card is Blue while the other 2 cards are Red.

Your goal is to identify the colors on the sides of the cards that you cannot see.

You know that one card is Red on both sides, one card is Blue on both sides, and one card is Red on one side and Blue on the other side.

You are allowed to select only one of the cards and observe the color of its other side.

Which card will you select to learn the colors of the other side of all three cards.

Describe your strategy and explain why it always works correctly.