CISC 2210 – Introduction to Discrete Structures

Midterm 1 Exam

October 6, 2022

Id: .....

Problem	Maximum Points	Your Points
Sets 1	15	
Sets 2	15	
Sets 3	20	
Logic 1	20	
Logic 2	20	
Logic 3	10	
Total	100	

Structure, problem selection, and credit:

- You have 90 minutes to complete the exam.
- There are two parts: one for the topic of Sets and one for the topic of Logic. Each part contains three problems. See above the credit that you can earn for each of the six problems for a total of 100 credits.
- You will get only partial credit if you fail to justify your answers. You will get 20% of the credit if you do not answer a problem. You will get zero credit for wrong answers.

Honor code: Students are expected to do this exam **by themselves** without any external help from other people, the Internet, books, or notes. Cheaters will be punished severely. At minimum, they will fail the exam, but they may fail the whole class. In addition, students who cheat risk disciplinary measures by Brooklyn College and CUNY.

1. **(15 credits)**

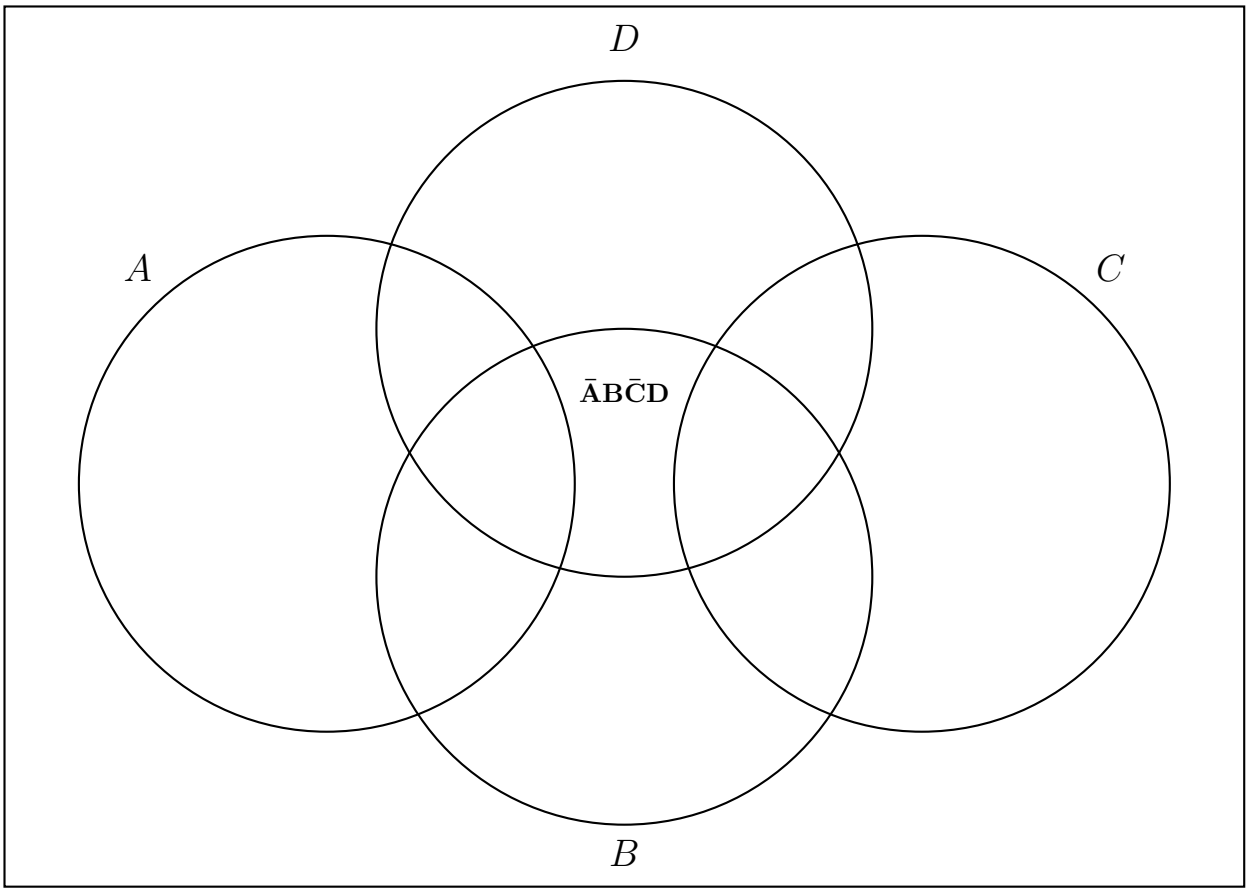
   Prove or disprove the following distributive law:

$$(A \cup B) \setminus C \equiv (A \setminus C) \cup (B \setminus C)$$

   If the law is correct provide a direct proof using membership arguments and if the law is not correct provide a counter example. In both cases, you may instead use Venn Diagrams as proofs.

2. **(15 credits)**

Below is a Venn Diagram for some sets A, B, C, and D.



(a) Mark all the existing zones as intersections of the sets or their complements. The zone $\bar{A}B\bar{C}D$ (stands for $\neg A \cap B \cap \neg C \cap D$) is already marked.
Note that in the diagram only 12 (out of 16) possible zones are represented.

(b) List the 4 zones that are not represented in this diagram.

3. **(20 credits)** Last week **36** students attended the discrete structures class: **14** students wore black shirts, **13** students wore black pants, and **12** students wore black shoes.

   5 students wore a black shirt and black pants, **3** students wore a black shirt and black shoes, **4** students wore black pants and black shoes, while only **1** student wore a black shirt, black pants, and black shoes.

   Justify your answers to the following four questions:

   (a) How many students did not wear a black shirt, black pants, or black shoes?

   (b) How many students wore a black shirt but did not wear black pants or black shoes?

   (c) How many students wore black pants but did not wear a black shirt or black shoes?

   (d) How many students wore black shoes but did not wear a black shirt or black pants?

4. **(20 credits)**

Define a boolean formula $\mathcal{P}$ on the variables x, y, and z for which the following table is its truth table.

Explain why your answer is correct.

Important: While, your main goal is to find a correct answer, to get the full credit, your optimization goal is to find as short as possible formula.

You may use any function you wish, however OR ($\vee$) and AND ($\wedge$)are enough.

x	y	z	$\mathcal{P}$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

5. **(20 credits)**

Let $X = \{1, 3, 5, 7, 9\}$, $Y = \{2, 4, 6, 8, 10\}$, and $Z = \{10, 11, 12, 13, 14\}$.

For each one of the following expressions, determine if it is TRUE or FALSE. Justify your answers.

Hint: Remember that order among quantifiers matters!

(a) $\forall_{x \in X} \forall_{y \in Y} \exists_{z \in Z} \{x + y < z\}$

(b) $\exists_{x \in X} \forall_{y \in Y} \exists_{z \in Z} \{x + y > z\}$

(c) $\exists_{z \in Z} \forall_{x \in X} \exists_{y \in Y} \{x + y = z\}$

6. **(10 credits)**

Yesterday, Alice, Bob, and Charlie raced against each other. The three racers placed first, second, and third (last).

- Alice claims to have arrived first;
- Bob claims that he did not arrive last; and
- Charlie claims that Alice arrived before Bob.

Justify your answers to the following three questions.

(a) In what order they finished the race if all of them are telling the truth?

(b) Could all of them be lying?

(c) If exactly one of them is lying while the other two are telling the truth, in what order they finished the race and who is the liar?