CISC 2210 – Introduction to Discrete Structures

Midterm 1 Exam

October 6, 2022

Id: ....

Problem	Maximum Points	Your Points
Sets 1	15	
Sets 2	15	
Sets 3	20	
Logic 1	20	
Logic 2	20	
Logic 3	10	
Total	100	

Structure, problem selection, and credit:

- You have 90 minutes to complete the exam.

- There are two parts: one for the topic of Sets and one for the topic of Logic. Each part contains three problems. See above the credit that you can earn for each of the six problems for a total of 100 credits.

- You will get only partial credit if you fail to justify your answers. You will get 20% of the credit if you do not answer a problem. You will get zero credit for wrong answers.

Honor code: Students are expected to do this exam **by themselves** without any external help from other people, the Internet, books, or notes. Cheaters will be punished severely. At minimum, they will fail the exam, but they may fail the whole class. In addition, students who cheat risk disciplinary measures by Brooklyn College and CUNY.
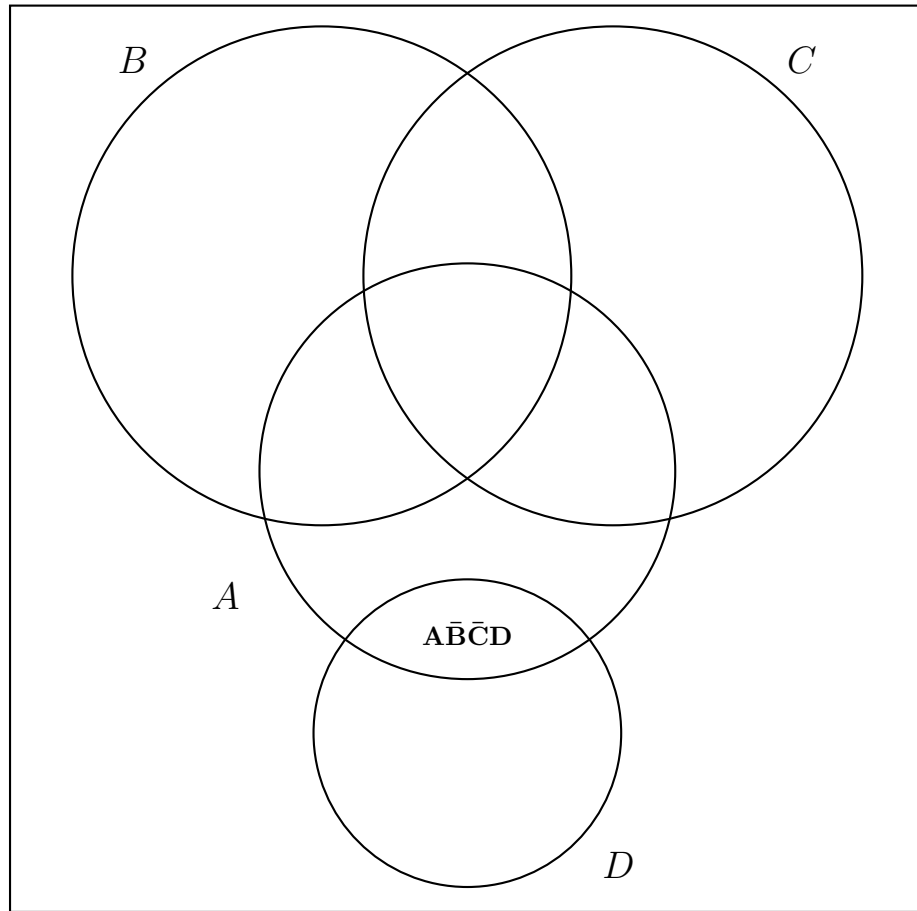
1. **(15 credits)**

   Prove or disprove the following distributive law:

   $$(A \cap B) \setminus C \equiv (A \setminus C) \cap (B \setminus C)$$

   If the law is correct provide a direct proof using membership arguments and if the law is not correct provide a counter example. In both cases, you may instead use Venn Diagrams as proofs.

2. **(15 credits)**

   Below is a Venn Diagram for some sets A, B, C, and D.

   B   C

   A

   A$\bar{\text{B}}\bar{\text{C}}$D

   D

   (a) Mark all the existing zones as intersections of the sets or their complements. The zone $A\bar{B}\bar{C}D$ (stands for $A \cap \neg B \cap \neg C \cap D$) is already marked.

      Note that in the diagram only 10 (out of 16) possible zones are represented.

   (b) List the 6 zones that are not represented in this diagram.

3. **(20 credits)**

The Computer Science department has **90** students: **40** students registered for the programming club, **38** students registered for the systems club, and **34** students registered for the math club.

13 students registered for both the programming club and the systems club, **18** students registered for both the programming club and the math club, **8** students registered for both the systems club and the math club, while **12** students registered for none of the clubs.

Justify your answers to the following four questions:

(a) How many students registered for all three clubs?

(b) How many students registered only for the programming club?

(c) How many students registered only for the systems club?

(d) How many students registered only for the math club?

4. **(20 credits)**

Define a boolean formula $\mathcal{P}$ on the variables x, y, and z for which the following table is its truth table.

Explain why your answer is correct.

Important: While, your main goal is to find a correct answer, to get the full credit, your optimization goal is to find as short as possible formula.

You may use any function you wish, however OR ($\lor$) and AND ($\land$) are enough.

x	y	z	$\mathcal{P}$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

5. **(20 credits)**

   Let $X = \{1, 2, 3, 4, 5\}$, $Y = \{1, 4, 9, 16, 25\}$, and $Z = \{1, 8, 27, 64, 125\}$.

   For each one of the following expressions, determine if it is TRUE or FALSE. Justify your answers.

   Hint: Remember that order among quantifiers matters!

   (a) $\forall_{x \in X} \exists_{y \in Y} \exists_{z \in Z} \{x \times y = z\}$

   (b) $\exists_{x \in X} \exists_{y \in Y} \forall_{z \in Z} \{x \times y \geq z\}$

   (c) $\forall_{x \in X} \exists_{y \in Y} \forall_{z \in Z} \{x \times y \leq z\}$

6. **(10 credits)**

Alice claims that today is Sunday. Bob claims that it is rainy outside. Charlie claims that if Alice is telling the truth then Bob is lying.

Charlie is lying.

What day is it today and is it rainy outside?

Justify your answer.

Hint: Recall the truth table of the boolean function $\mathcal{IMPLY}$ ($\rightarrow$).