CISC 2210 (TR11) – Introduction to Discrete Structures

Midterm 1 Exam

September 26, 2023

Id: ....

Problem	Maximum Points	Your Points
Sets 1	10	
Sets 2	20	
Sets 3	20	
Logic 1	20	
Logic 2	20	
Logic 3	10	
Total	100	

Structure and credit:

- You have 120 minutes to complete the exam.
- There are two parts: one for the topic of Sets and one for the topic of Logic. Each part contains three problems. See the chart above for the credit that you can earn for each of the six problems, for a total of 100 credits.
- You will get only partial credit if you fail to justify your answers. You will get 20% of the credit if you do not answer a problem. You will get zero credit for wrong answers.

Honor code: Students are expected to do this exam **by themselves** without any external help from other people, the Internet, books, or notes. Cheaters will be punished severely. At minimum, they will fail the exam, but they may fail the whole class. In addition, students who cheat risk disciplinary measures by Brooklyn College and CUNY.

1. Let A and B be two non-empty sets. Find the simplest form for each of the following four expressions involving these sets. Justify your answers.

   Hint: Three of these simplest forms contain only one set-operation while one of the simplest forms does not contain set-operations at all.

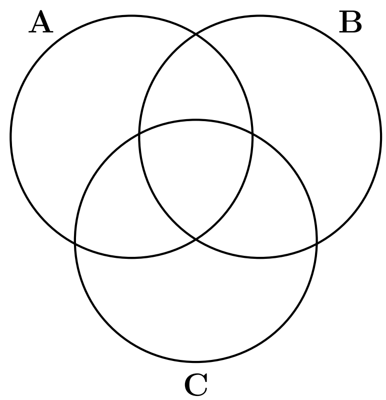
   (a) What is $A \cap (B \setminus A)$?

   (b) What is $A \cup (B \setminus A)$?

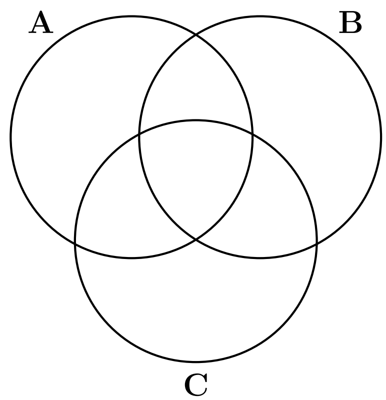
   (c) What is $A \setminus (A \cap B)$?

   (d) What is $(A \cup B) \setminus A$?

2. In the following Venn Diagram for the three sets A, B, and C, mark the zones that must be empty if $\mathbf{A} \cup \mathbf{C} = \mathbf{B} \cup \mathbf{C}$.



In the following Venn Diagram for the three sets A, B, and C, mark the zones that must be empty if $\mathbf{A} \cap \mathbf{C} = \mathbf{B} \cap \mathbf{C}$.



What is the relationship between the sets A and B if both $\mathbf{A} \cup \mathbf{C} = \mathbf{B} \cup \mathbf{C}$ and $\mathbf{A} \cap \mathbf{C} = \mathbf{B} \cap \mathbf{C}$? Justify your answer.

3. All of the students of the Brooklyn College CIS department won a free one week vacation in Europe to visit one or two out of the following four countries: England (**E**), France (**F**), Germany (**G**), and Italy (**I**). Students were not allowed to tour more than two countries.

   It so happened that among the students who took the tour to Europe, each one of the four countries was toured by exactly **40** students while each possible pair of countries was toured by exactly **10** students.

   Remark: There are six possible pairs of countries: $\{\mathbf{E,F}\}$, $\{\mathbf{E,G}\}$, $\{\mathbf{E,I}\}$, $\{\mathbf{F,G}\}$, $\{\mathbf{F,I}\}$, and $\{\mathbf{G,I}\}$.

   Justify your answers to the following two questions.

   (a) How many CIS students took the free vacation to Europe?

   (b) How many CIS students toured only England?

4. The following is the truth table of the function $\mathcal{XOR}$ (denoted by $\oplus$) with the boolean variables x and y.

x	y	$x \oplus y$
T	T	F
T	F	T
F	T	T
F	F	F

Prove that the following proposition with the three boolean variables x, y and z is a contradiction (that is, there is no truth assignment to this proposition):

$$(x \oplus y) \wedge (y \oplus z) \wedge (z \oplus x)$$

5. Let $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the set of all 10 digits.

The following eight boolean propositions include universal quantifiers, existential quantifiers, or their negations ($\forall$ or $\neg(\forall)$ or $\exists$ or $\neg(\exists)$).

Which **four** of them are TRUE and which **four** of them are FALSE?

Using the given space, you MUST provide explanations, examples, or counterexamples to support your answers. For each part, you will get

- 2.5 credits for a correct answer with a convincing explanation.
- 1.5 or 2 credits for a correct answer without a full explanation.
- 1 credit for a correct answer without any explanation.
- 0.5 credit for no answer.
- No credit for a correct answer with a wrong explanation.
- **-1 credit for a wrong answer**.

(a) $\forall_{x \in D} \forall_{y \in D} \, (x < y)$: ____

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(b) $\forall_{x \in D} \exists_{y \in D} \, (x < y)$: ____

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(c) $\exists_{x \in D} \forall_{y \in D} \, (x < y)$: ____

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(d) $\exists_{x \in D} \exists_{y \in D} \, (x < y)$: ____

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(e) $\forall_{x \in D} \neg(\forall_{y \in D} \, (x < y))$: ____________________________________

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(f) $\forall_{x \in D} \neg(\exists_{y \in D} \, (x < y))$: ____________________________________

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(g) $\exists_{x \in D} \neg(\forall_{y \in D} \, (x < y))$: ____________________________________

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(h) $\exists_{x \in D} \neg(\exists_{y \in D} \, (x < y))$: ____________________________________

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6. (a) A drawer contains **10** white socks and **10** black socks. Without looking, you blindly draw random socks from the drawer one at a time.

   Justify your answers to the following two questions.

   - What is the minimum number of socks you need to draw to guarantee that you have a pair of matching color socks?
   - What is the answer if the drawer contains **100** white socks and **100** black socks?

   (b) A drawer contains **10** identical pairs of black shoes (a right shoe and a left shoe). Without looking, you blindly draw random shoes from the drawer one at a time.

   Justify your answers to the following two questions.

   - What is the minimum number of shoes you need to draw to guarantee that you have at least one right shoe and at least one left shoe?
   - What is the answer if the drawer contains **100** identical pairs of black shoes?