CISC 2210 (TR2) – Introduction to Discrete Structures

Midterm 1 Exam

September 26, 2023

Id: ....

Problem	Maximum Points	Your Points
Sets 1	10	
Sets 2	20	
Sets 3	20	
Logic 1	20	
Logic 2	20	
Logic 3	10	
Total	100	

Structure and credit:

- You have 120 minutes to complete the exam.
- There are two parts: one for the topic of Sets and one for the topic of Logic. Each part contains three problems. See the chart above for the credit that you can earn for each of the six problems, for a total of 100 credits.
- You will get only partial credit if you fail to justify your answers. You will get 20% of the credit if you do not answer a problem. You will get zero credit for wrong answers.

Honor code: Students are expected to do this exam **by themselves** without any external help from other people, the Internet, books, or notes. Cheaters will be punished severely. At minimum, they will fail the exam, but they may fail the whole class. In addition, students who cheat risk disciplinary measures by Brooklyn College and CUNY.

1. Let A and B be two non-empty sets. Find the simplest form for each of the following four expressions involving these sets. Justify your answers.

   Hint: Two of these simplest forms contain only one set-operation while the other two simplest forms do not contain set-operations at all.

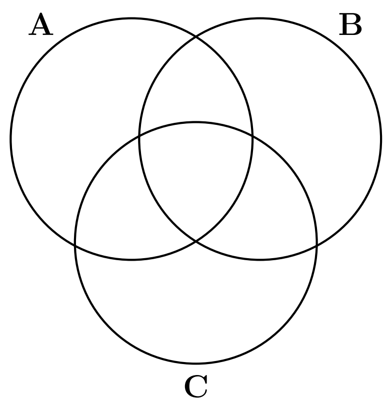
   (a) What is $(A \setminus B) \setminus A$?

   (b) What is $(A \setminus B) \setminus B$?

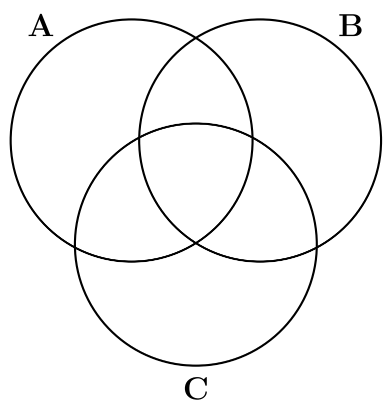
   (c) What is $A \setminus (B \setminus A)$?

   (d) What is $A \setminus (A \setminus B)$?

2. In the following Venn Diagram for the three sets A, B, and C, mark the zone that must be empty if $(\mathbf{A} \cap \mathbf{C}) \cap (\mathbf{B} \cap \mathbf{C}) = \emptyset$.



In the following Venn Diagram for the three sets A, B, and C, mark the zone that must be empty if $(\mathbf{A} \setminus \mathbf{C}) \cap (\mathbf{B} \setminus \mathbf{C}) = \emptyset$.



What can you say about the intersection $A \cap B$ if both $(\mathbf{A} \cap \mathbf{C}) \cap (\mathbf{B} \cap \mathbf{C}) = \emptyset$ and $(\mathbf{A} \setminus \mathbf{C}) \cap (\mathbf{B} \setminus \mathbf{C}) = \emptyset$? Justify your answer.

3. Many of the Brooklyn College students love playing a variety of sports. Each year, the college offers each student free practice sessions in at most two of the following four sports: Basketball (**B**), Soccer (**S**), Tennis (**T**), and Volleyball (**V**). Students are not allowed to select more than two sports.

   It so happened that among the students who chose to participate in the free practice sessions in 2023, each one of the four sports was selected by exactly **100** students while each possible pair of sports was selected by exactly **20** students.

   Remark: There are six possible pairs of sportss: {**B,S**}, {**B,T**}, {**B,V**}, {**S,T**}, {**S,V**}, and {**T,V**}.

   Justify your answers to the following two questions.

   (a) In 2023, how many students participated in at least one free practice session?

   (b) In 2023, how many students participated only in the Tennis practice session?

4. The following is the truth table of the function $\mathcal{EQUIV}$ (denoted by $\equiv$) with the boolean variables x and y.

x	y	$x \equiv y$
T	T	T
T	F	F
F	T	F
F	F	T

Prove that the following proposition with the three boolean variables x, y and z is a tautology (that is, all the eight possible assignments satisfy this proposition):

$$(x \equiv y) \vee (y \equiv z) \vee (z \equiv x)$$

5. Let $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the set of all 10 digits.

The following eight boolean propositions include universal quantifiers, existential quantifiers, or their negations ($\forall$ or $\neg(\forall)$ or $\exists$ or $\neg(\exists)$).

Which **four** of them are TRUE and which **four** of them are FALSE?

Using the given space, you MUST provide explanations, examples, or counterexamples to support your answers. For each part, you will get

- 2.5 credits for a correct answer with a convincing explanation.
- 1.5 or 2 credits for a correct answer without a full explanation.
- 1 credit for a correct answer without any explanation.
- 0.5 credit for no answer.
- No credit for a correct answer with a wrong explanation.
- **-1 credit for a wrong answer**.

(a) $\exists_{x \in D} \exists_{y \in D} (x > y)$: ______

(b) $\exists_{x \in D} \forall_{y \in D} (x > y)$: ______

(c) $\forall_{x \in D} \exists_{y \in D} (x > y)$: ______

(d) $\forall_{x \in D} \forall_{y \in D} (x > y)$: ______

(e) $\exists_{x \in D} \neg(\exists_{y \in D} (x > y))$: ______

(f) $\exists_{x \in D} \neg(\forall_{y \in D} (x > y))$: ______

(g) $\forall_{x \in D} \neg(\exists_{y \in D} (x > y))$: ______

(h) $\forall_{x \in D} \neg(\forall_{y \in D} (x > y))$: ______

6. (a) A box contains **7** blue balls and **7** red balls. Without looking, you blindly draw random balls from the box one at a time.

   Justify your answers to the following two questions.

   - What is the minimum number of balls you need to draw to guarantee that you have at least one blue ball and at least one red ball?
   - What is the answer if the box contains **25** blue balls and **25** red balls?

   (b) A box contains **7** blue balls and **7** red balls. Without looking, you blindly draw random balls from the box one at a time.

   Justify your answers to the following two questions.

   - What is the minimum number of balls you need to draw to guarantee that you have two balls of the same color?
   - What is the answer if the box contains **25** blue balls and **25** red balls?