CISC 2210 TR2 – Introduction to Discrete Structures

Midterm 2 Exam

Nov 7, 2023

Id: ....

Problem	Maximum Points	Your Points
1	100	
2	100	
3	100	
4	100	
5	100	

Structure, problem selection, and credit:

- You have 2 hours to complete the exam.
- There are 5 problems. Each problem is a "mini-exam" by itself with a 5% weight in the final grade for the class. However, the grade of each individual problem counts only if it is higher than the final exam grade.

  Strategy: It is better to try first answering the questions relating to topics you have mastered. Note that since there is no cumulative grade, one fully correct answer is better than two or more partially correct answers.
- You will get only partial credit if you fail to justify or prove your answers. You will get 20% of the credit for any problem or part of a problem if you leave the allocated space for the answer empty. You will get zero credit for wrong answers.

Honor code: Students are expected to do this exam **by themselves** without any external help from other people, the Internet, books, notes, or calculators. Cheaters will be punished severely. At minimum, they will fail the exam, but they may fail the whole class. In addition, students who cheat risk disciplinary measures by Brooklyn College and CUNY.

1. Prove the correctness of the following identity for any integer $n \geq 1$.
   You may use induction or any other method.

$$\sum_{i=1}^{n} i + \sum_{i=n+1}^{2n} i \; = \; n(2n+1)$$

2. Match each one of the following 5 recursive formulas with one of the possible 5 solutions:

$$-5n \quad ; \quad 5n \quad ; \quad 5n^2 \quad ; \quad 5^n \quad ; \quad 5n!$$

Justify your answers with few sentences.

(a) $T(1) = 5$; $T(n) = 5T(n-1)$ for $n > 1$

(b) $T(1) = -5$; $T(n) = T(n-1) - 5$ for $n > 1$

(c) $T(1) = 5$; $T(n) = T(n-1) + (10n - 5)$ for $n > 1$

(d) $T(1) = 5$; $T(n) = nT(n-1)$ for $n > 1$

(e) $T(1) = 5$; $T(n) = T(n-1) + 5$ for $n > 1$

3. A legal password of length 5 must contain exactly one of the three letters X, Y, Z while the other four positions must be filled with one of the ten digits $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$.

   Justify your answers to the following six questions.

   (a) How many legal passwords of length 5 are there in which the letter must appear in the first position?

   (b) How many legal passwords of length 5 are there in which the letter may appear in the first position or in the last position but cannot appear in the middle?

   (c) How many legal passwords of length 5 are there in which the letter may appear in any one of the five positions?

(d) How many legal passwords of length 5 are there in which all the four digits must be the same and the letter may appear in any of the five positions?

(e) How many legal passwords of length 5 are there in which all the four digits must be different and the letter must appear in the first position?

(f) How many legal passwords of length 5 are there in which two of the digits must be even while the other two digits must be odd and the letter must appear in the last position?

4. Prove the following identity for or any two integers k and n such that $1 \leq k \leq n$:

$$\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$$

5. You have three 3-sided fair dice. The faces of these dice are labeled with the numbers $1, 2, 3$. You throw the three dice together.

   Justify your answers to the following six questions.

   (a) What is the probability that the sum of the three shown numbers is 3?

   (b) What is the probability that the product of the three shown numbers is 4?

   (c) What is the probability that the product of the three shown numbers is odd?

   (d) What is the probability that the sum of the three shown numbers is even?

(e) Given that the product of the three shown numbers is larger than 7, what is the probability that their sum is exactly 7?

(f) Given that the sum of the three shown numbers is 4 or 5, what is the probability that their product is exactly 4?