CISC 2210 (TR11) – Introduction to Discrete Structures

Midterm 1 Exam

September 25, 2025

Id: ....

Problem	Maximum Points	Your Points
Sets 1	10	
Sets 2	20	
Sets 3	20	
Logic 1	20	
Logic 2	20	
Logic 3	10	
Total	100	

Structure and credit:

- You have 120 minutes to complete the exam.
- There are two parts: one for the topic of Sets and one for the topic of Logic. Each part contains three problems. See the chart above for the credit that you can earn for each of the six problems, for a total of 100 credits.
- You will get only partial credit if you fail to justify your answers. You will get 20% of the credit if you do not answer a problem. You will get zero credit for wrong answers.

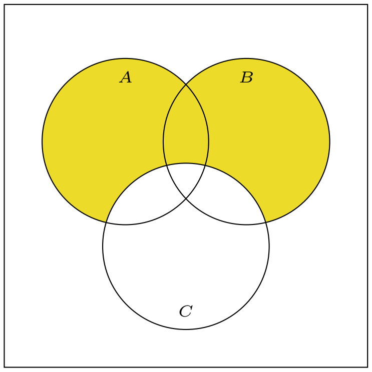
Honor code: Students are expected to do this exam **by themselves** without any external help from other people, the Internet, books, or notes. Cheaters will be punished severely. At minimum, they will fail the exam, but they may fail the whole class. In addition, students who cheat risk disciplinary measures by Brooklyn College and CUNY.

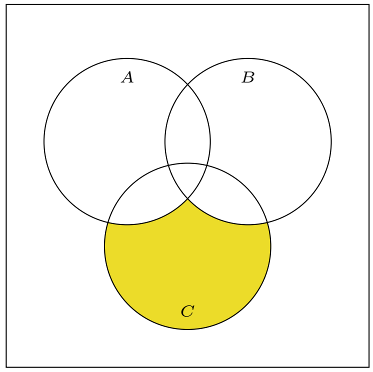
1. Let A, B, and C be three sets whose members are taken from a universal set $\mathcal{U}$. Justify your answers to the following two questions.

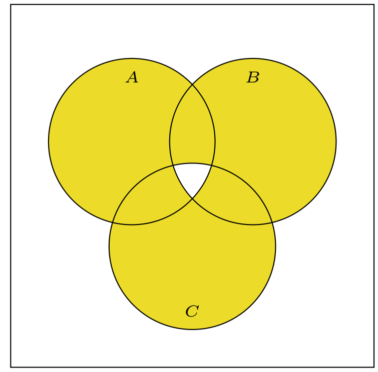
Assume $(\neg A) \cap (\neg B) \cap (\neg C) = \emptyset$. What is $A \cup B \cup C$?

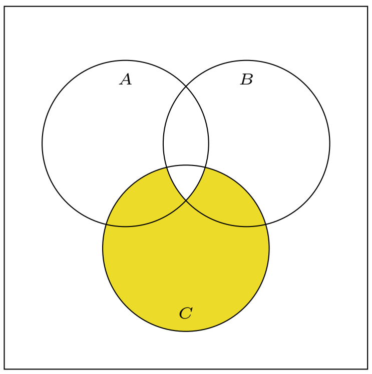
Assume $(\neg A) \cup (\neg B) \cup (\neg C) = \emptyset$. What can you say about the sets A, B, and C?

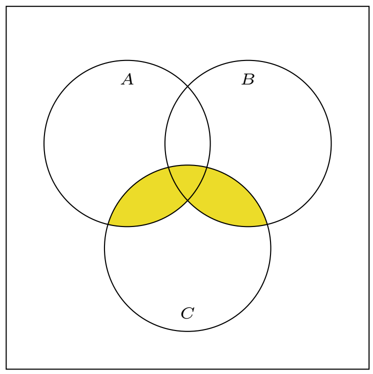
2. Match the four expressions on the next page with their corresponding Venn diagrams from the 12 provided below. Justify your choices.

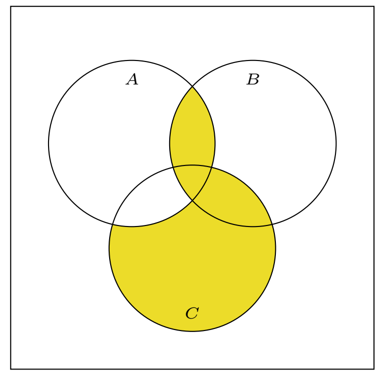


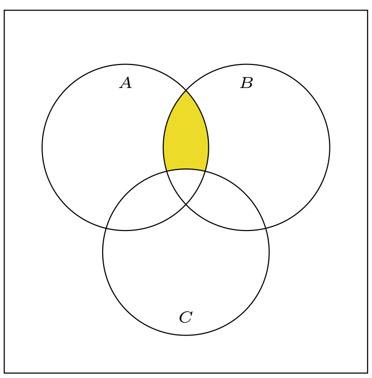


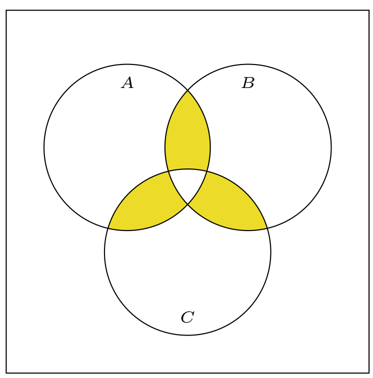


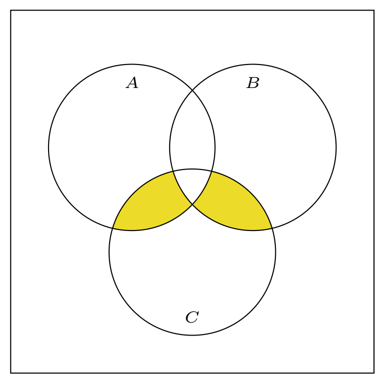


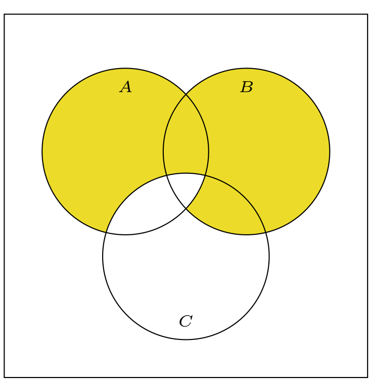


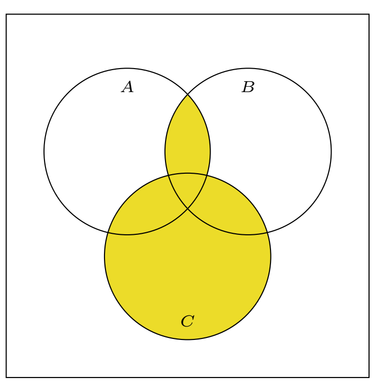


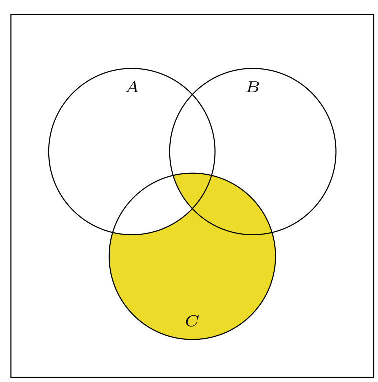












Which Venn Diagram represents the expression: $(\mathbf{A} \setminus \mathbf{C}) \cap (\mathbf{B} \setminus \mathbf{C})$?

Which Venn Diagram represents the expression: $(\mathbf{C} \setminus \mathbf{A}) \cap (\mathbf{C} \setminus \mathbf{B})$?

Which Venn Diagram represents the expression: $\mathbf{C} \setminus (\mathbf{A} \setminus \mathbf{B})$?

Which Venn Diagram represents the expression: $(\mathbf{C} \setminus \mathbf{A}) \cup (\mathbf{A} \cap \mathbf{B})$?

3. At a cat show, every cat wore at least one ribbon. The available colors were Blue, Red, and Green. **Based on the data below, determine the total number of cats at the show.**
   - Every cat wore ribbons of at least two different colors.
   - 80 cats wore both Blue and Red ribbons (and maybe also a Green ribbon).
   - 60 cats wore both Red and Green ribbons (and maybe also a Blue ribbon).
   - 40 cats wore both Green and Blue ribbon (and maybe also a Red ribbon).
   - 20 cats wore ribbons of all three colors.

   Justify your answer.

At a dog show, every dog wore at least one ribbon. The available colors were Blue, Red, and Green. **Based on the data below, determine how many dogs wore ribbons of all three colors.**

- There were 100 dogs in total.
- No dog wore ribbons of exactly two different colors.
- 50 dogs wore a Blue ribbon (and maybe also a Red ribbon and a Green ribbon).
- 40 dogs wore a Red ribbon (and maybe also a Blue ribbon and a Green ribbon).
- 30 dogs wore a Green ribbon (and maybe also a Blue ribbon and a Red ribbon).

Justify your answer.

4. One of the following two expressions is True and one is False.

$$x \vee \neg(y \vee z) \;\equiv\; x \wedge \neg(y \wedge z) \tag{1}$$

$$(x \wedge \neg y) \vee (\neg x \wedge z) \equiv \; (x \vee z) \wedge (\neg x \vee \neg y) \tag{2}$$

Which one is False? Prove your answer.

Which one is True? Prove your answer.

5. The table below shows which sports are played by seven students. A T in the table means the student in that column plays the sport in that row. For example, the T in the (Basketball,Bob) entry means Bob plays Basketball while the F in the (Football,Carol) entry means Carol does not play Football.

Sport \ Student	Alice	Bob	Carol	David	Eva	Frank	Gina
Basketball	F	T	T	T	F	T	F
Football	T	T	F	F	T	F	F
Tennis	F	T	T	T	T	F	F

Let X be the set containing the 7 students and let Y be the set containing the 3 sports:

$$X = \{\text{Alice}, \text{Bob}, \text{Carol}, \text{David}, \text{Eva}, \text{Frank}, \text{Gina}\} \quad Y = \{\text{Basketball}, \text{Football}, \text{Tennis}\}$$

For each one of the following expressions determine if it is TRUE or FALSE. Justify your answers.

(a) $\forall_{x \in X} \forall_{y \in Y} \, (x \text{ plays } y)$

(b) $\exists_{x \in X} \exists_{y \in Y} \, (x \text{ plays } y)$

(c) $\forall_{x \in X} \exists_{y \in Y} \, (x \text{ plays } y)$

(d) $\forall_{y \in Y} \exists_{x \in X} \, (x \text{ plays } y)$

(e) $\exists_{x \in X} \forall_{y \in Y} \, (x \text{ plays } y)$

(f) $\exists_{y \in Y} \forall_{x \in X} \, (x \text{ plays } y)$

6. You meet A, B, and C. One of them is a truth-teller who always tells the truth, one of them is a liar who always lies, and one of them is a mixer who sometimes tells the truth and sometimes lies.

   - A speaks first: “I am not a truth-teller”.
   - B speaks second: “I am not a mixer”.
   - C is silent.

   Who is the truth-teller? Who is the liar? And who is the mixer?

   Justify your answer.