CISC 2210 TR11 – Introduction to Discrete Structures

Midterm 2 Exam

Nov 11, 2025

Id: ....

Problem	Maximum Points	Your Points
Induction	100	
Recursion	100	
Counting	100	
Combinatorics	100	
Probability	100	

Structure, problem selection, and credit:

- You have 2 hours to complete the exam.
- There are 5 problems. Each problem is a "mini-exam" by itself with a 5% weight in the final grade for the class. However, the grade of each individual problem counts only if it is higher than the final exam grade.

  Strategy: It is better to try first answering the questions relating to topics you have mastered. Note that since there is no cumulative grade, one fully correct answer is better than two or more partially correct answers.
- You will get only partial credit if you fail to justify or prove your answers. You will get 20% of the credit for any problem or part of a problem if you leave the allocated space for the answer empty. You will get zero credit for wrong answers.

Honor code: Students are expected to do this exam **by themselves** without any external help from other people, the Internet, books, notes, or calculators. Cheaters will be punished severely. At minimum, they will fail the exam, but they may fail the whole class. In addition, students who cheat risk disciplinary measures by Brooklyn College and CUNY.

Problem 1: Induction (100 credits)

Prove by induction the following identity for all integers $n \geq 2$:

$$1+2+3+\cdots+(n-2)+(n-1)+n+(n-1)+(n-2)+\cdots+3+2+1 = n^2$$

In other words, using induction, prove for all integers $n \geq 2$ that n^2 is the sum of the positive integers ascending from 1 to n and then descending back to 1.

Problem 2: Recursion (100 credits)

Define the following recursive formula $T(n)$ for all nonnegative integers $n \geq 0$:

$$T(n) = \begin{cases} 1 & \text{for } n = 0 \\ 3 & \text{for } n = 1 \\ 2T(n-1) + 3T(n-2) & \text{for } n \geq 2 \end{cases}$$

Problem 2a (50 credits)

Find a closed-form expression for $T(n)$. Justify your answer.

Problem 2b (50 credits)

Prove that the expression from part (a) is correct. Note that bottom-up or top-down evaluations will not be considered valid proofs.

Problem 3: Counting (100 credits)

A ternary string is a list with repetitions $(d_1, d_2, \ldots, d_n)$ such that $d_i \in \{0, 1, 2\}$ for all $1 \le i \le n$. For example, 11021 is a length-5 ternary string, 0010 is a length-4 ternary string, and 22 is a length-2 ternary string.

Justify your answers to the following six questions.

Problem 3a (10 credits)

For $n \ge 1$, how many length-n ternary strings are there?

Problem 3b (20 credits)

For $n \ge 2$, in how many length-n ternary strings $d_1 = d_n$?

Problem 3c (20 credits)

For $n \ge 2$, in how many length-n ternary strings $d_1 \ne d_n$?

Problem 3d (20 credits)

For $n \geq 1$, how many length-n ternary strings do not contain 0?

Problem 3e (20 credits)

For $n \geq 1$, how many length-n ternary strings contain at least one 0?

Problem 3f (10 credits)

For $n \geq 2$, in how many length-n ternary strings $d_i \neq d_{i-1}$ for all $2 \leq i \leq n$?

Problem 4: Combinatorics (100 credits)

A 10-member club needs to form two separate 3-person committees.

Justify your answers for parts (a), (b), and (c). In your answers you do not need to evaluate binomial coefficients. You may leave them as $\binom{n}{k}$ for some n and k.

Problem 4a (40 credits)

How many ways can the club select the two committees if members are allowed to be on both?

Problem 4b (40 credits)

How many ways can the club select the two committees if the committees must be disjoint (meaning no member can be on both)?

Problem 4c (20 credits)

How many ways can the club select the two committees if they must share exactly one common member?

Problem 5: Probability (100 credits)

Bag A contains 3 yellow and 9 red marbles, while Bag B contains 6 yellow and 6 blue marbles. A 6-sided fair die is rolled to determine which bag to draw from: if the die shows a 5 or 6 (a 1/3 probability), a single marble is blindly drawn from Bag A; otherwise (a 2/3 probability), a single marble is blindly drawn from Bag B. Justify your answers for Part (a) and Part (b).

Problem 5a (50 credits)

What is the probability that the drawn marble is yellow?

Problem 5b (50 credits)

What is the probability that Bag A was the chosen bag, given that the drawn marble was yellow?