CISC 2210 – Introduction to Discrete Structures

Midterm 1 Exam

Mar 8, 2022

Id: ....

Problem	Maximum Points	Your Points
Sets 1	15	
Sets 2	15	
Sets 3	20	
Logic 1	20	
Logic 2	20	
Logic 3	10	
Total	100	

Structure, problem selection, and credit:

- You have 75 minutes to complete the exam.
- There are two parts: one for the topic of Sets and one for the topic of Logic. Each part contains three problems. See above the credit that you can earn for each of the six problems for a total of 100 credits.
- You will get only partial credit if you fail to justify your answers. You will get 20% of the credit if you do not answer a problem. You will get zero credit for wrong answers.

Honor code: Students are expected to do this exam **by themselves** without any external help from other people, the Internet, books, or notes. Cheaters will be punished severely. At minimum, they will fail the exam, but they may fail the whole class. In addition, students who cheat risk disciplinary measures by Brooklyn College and CUNY.

1. **(15 credits)**

   Let $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the set of all ten digits.

   (a) Define two proper subsets of D, $A \subset D$ and $B \subset D$, with the following properties:

   - The union of both sets is D: $A \cup B = D$.
   - The intersection of both sets contains two digits: $|A \cap B| = 2$.

   (b) Prove that there are no two subsets of D, $A \subseteq D$ and $B \subseteq D$, with the following properties:

   - The intersection of both sets is not empty: $A \cap B \neq \emptyset$.
   - The sizes of A and B are different: $|A| \neq |B|$.
   - The sizes of $A \setminus B$ and $B \setminus A$ are the same: $|A \setminus B| = |B \setminus A|$.

2. **(15 credits)**

Let A, B, and C, be three non-empty sets. Consider the following three sets:

$$\begin{aligned} R &= (A \cap \overline{B} \cap \overline{C}) \cup (\overline{A} \cap B \cap \overline{C}) \cup (\overline{A} \cap \overline{B} \cap C) \\ S &= (A \cup B \cup C) \setminus (A \cap B \cap C) \\ T &= (A \setminus (B \cup C)) \cup (B \setminus (A \cup C)) \cup (C \setminus (A \cup B)) \end{aligned}$$

Which two of of these sets are identical?

Explain why the third set is different from the two identical sets.

3. **(20 credits)**

Let A, B, and C be three non empty sets.

(a) Draw the Venn-diagram for these sets in which C is a proper subset of the intersection between A and B ($C \subset (A \cap B)$).

(b) What are the sizes of A, B, and $A \cup B$ given the following data:

- $|C| = 3$
- $|A \cap B| = 5$
- $|A \setminus C| = 10$
- $|B \setminus C| = 12$

Justify your answers.

4. **(20 credits)**

Let $S = \{0, 2, 4, 6, 8\}$ be the set of the five even digits and Let $T = \{1, 3, 5, 7, 9\}$ be the set of the five odd digits.

For each one of the following expressions, determine if it is TRUE or FALSE. Justify your answers.

(a) $\forall_{x \in S} \forall_{y \in T} \, (y = x + 1)$

(b) $\exists_{x \in S} \exists_{y \in T} \, (y = x + 1)$

(c) $\forall_{x \in S} \exists_{y \in T} \, (y = x + 1)$

(d) $\forall_{y \in T} \exists_{x \in S} \, (y = x + 1)$

(e) $\exists_{x \in S} \forall_{y \in T} \, (y = x + 1)$

(f) $\exists_{y \in T} \forall_{x \in S} \, (y = x + 1)$

5. **(20 credits)**

The goal is to express the operator $\mathcal{XOR}$ with the operators $\mathcal{AND}$, $\mathcal{OR}$, and $\mathcal{NOT}$. The truth table for the operator $\mathcal{XOR}$ is:

x	y	$x \oplus y$
T	T	F
T	F	T
F	T	T
F	F	F

(a) Express $\mathcal{XOR}$ ($\oplus$) with $\mathcal{AND}$ ($\wedge$), $\mathcal{OR}$ ($\vee$), and $\mathcal{NOT}$ ($\neg$). Justify your answer.

(b) Express $\mathcal{XOR}$ ($\oplus$) with $\mathcal{AND}$ ($\wedge$) and $\mathcal{NOT}$ ($\neg$). In this part, you cannot use $\mathcal{OR}$ ($\vee$). Justify your answer.

Hint: One way to do this is to apply one of the De Morgan's laws on your answer to part (a).

6. **(10 credits)**

You interrogate three people: A, B, and C. One of them stole your wallet.

- A claims that B stole the wallet.
- B claims that A stole the wallet.
- C agrees with B that A stole the wallet.

Justify your answers to the following two questions.

(a) Who stole the wallet if only one of them lying while the other two are telling the truth?

(b) Who stole the wallet if only one of them is telling the truth while the other two are lying?