CISC 2210 – Introduction to Discrete Structures

Midterm 1 Exam

Mar 8, 2022

Id: ....

Problem	Maximum Points	Your Points
Sets 1	15	
Sets 2	15	
Sets 3	20	
Logic 1	20	
Logic 2	20	
Logic 3	10	
Total	100	

Structure, problem selection, and credit:

- You have 75 minutes to complete the exam.
- There are two parts: one for the topic of Sets and one for the topic of Logic. Each part contains three problems. See above the credit that you can earn for each of the six problems for a total of 100 credits.
- You will get only partial credit if you fail to justify your answers. You will get 20% of the credit if you do not answer a problem. You will get zero credit for wrong answers.

Honor code: Students are expected to do this exam **by themselves** without any external help from other people, the Internet, books, or notes. Cheaters will be punished severely. At minimum, they will fail the exam, but they may fail the whole class. In addition, students who cheat risk disciplinary measures by Brooklyn College and CUNY.

1. **(15 credits)**

Let $S = \{0, 2, 4, 6, 8\}$ be the set of the even digits and let $T = \{1, 3, 5, 7, 9\}$ be the set of the odd digits.

(a) Define a subset A of the set of all digits with the following properties:

- A contains at least one odd digit and at least one even digit: $A \cap S \neq \emptyset$ and $A \cap T \neq \emptyset$.
- The intersection of A with S contains two more digits than the intersection of A with T: $|S \cap A| = |T \cap A| + 2$.

(b) Prove that there are no subsets $A \subseteq S$ and $B \subseteq T$ with the following properties:

- Both sets are not empty: $A \neq \emptyset$ and $B \neq \emptyset$.
- The intersection of A and B is not empty: $A \cap B \neq \emptyset$.

2. **(15 credits)**

Let A, B, and C, be three non-empty sets. Consider the following three sets:

$$\begin{aligned} R &= A \cap (\overline{B \cap C}) \\ S &= (A \cup B) \cap (A \cup C) \\ T &= A \setminus (A \cap B \cap C) \end{aligned}$$

Which two of of these sets are identical?

Explain why the third set is different from the two identical sets.

3. **(20 credits)**

Let A, B, C, and D be four non empty sets.

(a) Draw the Venn-diagram for these sets in which C is a proper subset of $A \setminus B$ ($C \subset (A \setminus B)$) and D is a proper subset of $B \setminus A$ ($D \subset (B \setminus A)$).

(b) What are the sizes of A, B, and $A \cap B$ given the following data:

- $|C| = 3$
- $|D| = 5$
- $|A \setminus B| = 10$
- $|B \setminus A| = 13$
- $|A \cup B| = 30$

Justify your answers.

4. **(20 credits)**

The following table summarizes which of the 5 colors Green, Magenta, Purple, Red, and Yellow are liked by the 5 students Alice, Bob, Carole, David, and Eva. If the value of an entry in the matrix is T, then the student from the entry's row likes the color from the entry's column, otherwise the student does not like the color.

For example, Bob likes Red because the value of the (Bob,Red) entry in the matrix is T while Eva does not like Yellow because the value of the (Eva,Yellow) entry in the matrix is F.

Student \ Colors	Green	Magenta	Purple	Red	Yellow
Alice	T	F	F	F	T
Bob	F	T	T	T	F
Carol	F	F	T	F	F
David	T	T	T	T	T
Eva	F	T	T	F	F

Let $S = \{\text{Alice}, \text{Bob}, \text{Carole}, \text{David}, \text{Eva}\}$ be the set containing the five students and let $C = \{\text{Green}, \text{Magenta}, \text{Purple}, \text{Red}, \text{Yellow}\}$ be the set containing the five colors.

For each one of the following expressions determine if it is TRUE or FALSE. Justify your answers.

(a) $\forall_{x \in S} \forall_{y \in C}\ (x \text{ likes } y)$

(b) $\exists_{x \in S} \exists_{y \in C}\ (x \text{ likes } y)$

(c) $\forall_{x \in S} \exists_{y \in C} \, (x \text{ likes } y)$

(d) $\forall_{y \in C} \exists_{x \in S} \, (x \text{ likes } y)$

(e) $\exists_{x \in S} \forall_{y \in C} \, (x \text{ likes } y)$

(f) $\exists_{y \in C} \forall_{x \in S} \, (x \text{ likes } y)$

5. **(20 credits)**

The goal is to express the operator $\mathcal{NAND}$ with the operators $\mathcal{OR}$ and $\mathcal{NOT}$ and the operator $\mathcal{NOR}$ with the operators $\mathcal{AND}$ and $\mathcal{NOT}$. The truth tables for the operators $\mathcal{NAND}$ ($\uparrow$) and $\mathcal{NOR}$ ($\downarrow$) are:

x	y	$x \uparrow y$
T	T	F
T	F	T
F	T	T
F	F	T

x	y	$x \downarrow y$
T	T	F
T	F	F
F	T	F
F	F	T

(a) Express $\mathcal{NAND}$ ($\uparrow$) only with $\mathcal{OR}$ ($\vee$) and $\mathcal{NOT}$ ($\neg$). In this part, you cannot use $\mathcal{AND}$ ($\wedge$). Justify your answer.

(b) Express $\mathcal{NOR}$ ($\downarrow$) only with $\mathcal{AND}$ ($\wedge$) and $\mathcal{NOT}$ ($\neg$). In this part, you cannot use $\mathcal{OR}$ ($\vee$). Justify your answer.

6. **(10 credits)**

   You are visiting an island whose people are either always telling the truth (knights) or always lying (knaves).

   Explain why you cannot hear the following conversation between person A and person B:

   - A shouts at B that B is a knave.
   - B answers that A is right.