CISC 2210 – Introduction to Discrete Structures

Makeup Midterm Exam 2

April 25, 2023

Id: .....

Problem	Maximum Points	Your Points
1	100	
2	100	
3	100	
4	100	
5	100	

Structure, problem selection, and credit:

- You have 90 minutes to complete the exam.
- There are 5 problems. Each problem is a "mini-exam" by itself with a 5% weight in the final grade for the class. However, the grade of each individual problem counts only if it is higher than the final exam grade.

  Strategy: It is better to try first answering the questions relating to topics you have mastered. Note that since there is no cumulative grade, one fully correct answer is better than two or more partially correct answers.
- You will get only partial credit if you fail to justify or prove your answers. You will get 20% of the credit for any problem or part of a problem if you leave the allocated space for the answer empty. You will get zero credit for wrong answers.

Honor code: Students are expected to do this exam **by themselves** without any external help from other people, the Internet, books, notes, or calculators. Cheaters will be punished severely. At minimum, they will fail the exam, but they may fail the whole class. In addition, students who cheat risk disciplinary measures by Brooklyn College and CUNY.

1. Prove the correctness of the following identity for any $n \geq 1$.

   You may use induction or any other method.

$$\sum_{i=1}^{n} i + \sum_{i=1}^{n} 2i + \sum_{i=1}^{n} 3i \;=\; 3n(n+1)$$

2. Consider the following recurrence for integers $n \geq 1$:

$$M(n) = \begin{cases} 3 & \text{for } n = 1 \\ 2M(n-1) + 3 & \text{for } n \geq 2 \end{cases}$$

Prove that for $n \geq 1$

$$M(n) = 3(2^n - 1)$$

Remark: The top-down evaluation and the bottom-up evaluation are not considered as proofs.

3. An ice cream shop sells several flavors of ice cream. You plan to buy three scoops in a cone and you **care about their order**. You might or might not buy a particular flavor more than once.

   Justify your answers to the following questions.

   Part (a): There are only 4 flavors.

   Find the number of 3-scoop cones you can buy.

   Find the number of 3-scoop cones you can buy in which the bottom scoop must be chocolate.

   Find the number of 3-scoop cones you can buy in which the bottom scoop cannot be chocolate.

   How do the three questions above relate to each other?

Part (b): There are n flavors for an integer $n \geq 3$.

Find the number of 3-scoop cones you can buy.

Find the number of 3-scoop cones you can buy in which the bottom scoop must be chocolate.

Find the number of 3-scoop cones you can buy in which the bottom scoop cannot be chocolate.

How do the three questions above relate to each other?

4. Simplify the following expression into an expression that does not contain binomial coefficients, factorials, and fractions.

   Explain how you found the simplified expression.

$$\binom{n+1}{2} - \binom{n-1}{2} + 1$$

5. A bag contains 6 marbles: 1 Blue (B) marble, 2 Red (R) marbles, and 3 Green (G) marbles. Two random marbles are drawn from the bag.

   Justify your answers to the following four questions.

   Part (a): What is the probability that both marbles are of the same color?

   Part (b): What is the probability that both marbles are of different colors?

Part (c): What is the probability that both marbles are of the same color given that at least one of them is Green?

Part (d): What is the probability that both marbles are of different colors given that at least one of them is Red?