CISC 2210 TR11 – Introduction to Discrete Structures

Midterm Exam 2

April 18, 2023

Id: .....

Problem	Maximum Points	Your Points
1	100	
2	100	
3	100	
4	100	
5	100	

Structure, problem selection, and credit:

- You have 90 minutes to complete the exam.
- There are 5 problems. Each problem is a "mini-exam" by itself with a 5% weight in the final grade for the class. However, the grade of each individual problem counts only if it is higher than the final exam grade.

  Strategy: It is better to try first answering the questions relating to topics you have mastered. Note that since there is no cumulative grade, one fully correct answer is better than two or more partially correct answers.
- You will get only partial credit if you fail to justify or prove your answers. You will get 20% of the credit for any problem or part of a problem if you leave the allocated space for the answer empty. You will get zero credit for wrong answers.

Honor code: Students are expected to do this exam **by themselves** without any external help from other people, the Internet, books, notes, or calculators. Cheaters will be punished severely. At minimum, they will fail the exam, but they may fail the whole class. In addition, students who cheat risk disciplinary measures by Brooklyn College and CUNY.

1. Prove the correctness of the following identity for any $n \geq 1$.

   You may use induction or any other method.

$$\sum_{i=1}^{n}(3i+1) - \sum_{i=1}^{n}(i+1) \;=\; n(n+1)$$

2. Consider the following recurrence for integers $n \geq 1$:

$$M(n) = \begin{cases} 1 & \text{for } n = 1 \\ 3M(n-1) + 1 & \text{for } n \geq 2 \end{cases}$$

Prove that for $n \geq 1$

$$M(n) = \frac{3^n - 1}{2}$$

Remark: The top-down evaluation and the bottom-up evaluation are not considered as proofs.

3. You are ordering a pizza. You plan to add toppings (**order does not matter**) chosen from a list of toppings that contains pineapple as one of the options.

   Justify your answers to the following questions.

   Part (a): You want 3 **different** toppings chosen from a list of 5 toppings.

   How many topping combinations do you have for your pizza?

   How many topping combinations do you have for your pizza if you would never consider pineapple as one of your toppings?

   How many topping combinations do you have for your pizza if you insist on having pineapple as one of your toppings?

   How do the three questions above relate to each other?

part (b): You want k **different** toppings chosen from a list of n toppings for some integers $n > k \geq 1$ in which one of the toppings is pineapple.

How many topping combinations do you have for your pizza?

How many topping combinations do you have for your pizza if you would never consider pineapple as one of your toppings?

How many topping combinations do you have for your pizza if you insist on having pineapple as one of your toppings?

How do the three questions above relate to each other?

4. Simplify the following expression into an expression that does not contain binomial coefficients, factorials, and fractions.

   Explain how you found the simplified expression.

$$\binom{n}{2} + \binom{n-1}{2}$$

5. Two **fair** dice are thrown: one is a 5**-sided** dice labeled with the numbers $1, 2, 3, 4, 5$ on its 5 faces and one is a 4**-sided** dice labeled with the numbers $1, 2, 3, 4$ on its 4 faces.

   Justify your answers to the following four questions.

   Part (a): What is the probability that the **sum** of the two shown numbers is **even**?

   Part (b): What is the probability that the **product** of the two shown numbers is **even**?

Part (c): What is the probability that the **product** of the two shown numbers is **even** given that their **sum** is **even**?

Part (d): What is the probability that the **sum** of the two shown numbers is **even** given that their **product** is **even**?