CISC 2210 TR2– Introduction to Discrete Structures

Midterm Exam 2

April 18, 2023

Id: .....

Problem	Maximum Points	Your Points
1	100	
2	100	
3	100	
4	100	
5	100	

Structure, problem selection, and credit:

- You have 90 minutes to complete the exam.
- There are 5 problems. Each problem is a "mini-exam" by itself with a 5% weight in the final grade for the class. However, the grade of each individual problem counts only if it is higher than the final exam grade.

  Strategy: It is better to try first answering the questions relating to topics you have mastered. Note that since there is no cumulative grade, one fully correct answer is better than two or more partially correct answers.
- You will get only partial credit if you fail to justify or prove your answers. You will get 20% of the credit for any problem or part of a problem if you leave the allocated space for the answer empty. You will get zero credit for wrong answers.

Honor code: Students are expected to do this exam **by themselves** without any external help from other people, the Internet, books, notes, or calculators. Cheaters will be punished severely. At minimum, they will fail the exam, but they may fail the whole class. In addition, students who cheat risk disciplinary measures by Brooklyn College and CUNY.

1. Prove the correctness of the following identity for any $n \geq 1$.

   You may use induction or any other method.

$$\sum_{i=1}^{n}(i-1) + \sum_{i=1}^{n}(i+1) \;=\; n(n+1)$$

2. Consider the following recurrence for integers $n \geq 1$:

$$M(n) = \begin{cases} 2 & \text{for } n = 1 \\ 3M(n-1) - 1 & \text{for } n \geq 2 \end{cases}$$

Prove that for $n \geq 1$

$$M(n) = \frac{3^n + 1}{2}$$

Remark: The top-down evaluation and the bottom-up evaluation are not considered as proofs.

3. A box contains some blue socks, some green socks, and some red socks.
   Justify your answers to the following questions.
   Part (a): There are 4 blue socks, 3 green socks, and 2 red socks in the box.
   Find the number of ways 2 socks can be drawn from the box.

Find the number of ways 2 socks of the same color can be drawn from the box.

Find the number of ways 2 socks of different colors can be drawn from the box.

How do the three questions above relate to each other?

Part (b): There are n socks in the box out of which b are blue socks, g are green socks, and r are red socks for some three integers $b \geq 2$, $g \geq 2$, and $r \geq 2$ such that $n = b + g + r$.

Find the number of ways 2 socks can be drawn from the box.

Find the number of ways 2 socks of the same color can be drawn from the box.

Find the number of ways 2 socks of different colors can be drawn from the box.

How do the three questions above relate to each other?

4. Simplify the following expression into an expression that does not contain binomial coefficients, factorials, and fractions.

   Explain how you found the simplified expression.

$$\binom{n+2}{2} - \binom{n}{2}$$

5. Three **fair** Heads and Tails coins are flipped.

   Justify your answers to the following four questions.

   Part (a): What is the probability that all the coins show Heads?

   Part (b): What is the probability that at least one coin shows Tails?

Part (c): What is the probability that all the coins show Heads given that at least one of the coins shows Heads?

Part (d): What is the probability that at least one coin shows Tails given that at least one of the coins shows Heads?