CISC 2210 (TR2) – Introduction to Discrete Structures

Midterm 1 Exam

March 3, 2026

Id: ....

Problem	Maximum Points	Your Points
Sets 1	20	
Sets 2	40	
Sets 3	40	
Sets Total	100	
Logic 1	40	
Logic 2	40	
Logic 3	20	
Logic Total	100	

Structure and credit:

- You have 120 minutes to complete the exam.
- There are two parts to this exam: Sets and Logic, with three problems per part. The credit for each problem is listed in the chart above. Each part has a maximum score of 100 credits.
- You will get only partial credit if you fail to justify your answers. You will get 20% of the credit if you do not answer a problem. You will get zero credit for wrong answers.

Honor code: Students are expected to do this exam **by themselves** without any external help from other people, the Internet, books, or notes. Cheaters will be punished severely. At minimum, they will fail the exam, but they may fail the whole class. In addition, students who cheat risk disciplinary measures by Brooklyn College and CUNY.

1. Prove or disprove: For any three sets A, B, and C,

$$C \setminus (A \cup B) = (C \setminus A) \cap (C \setminus B)$$

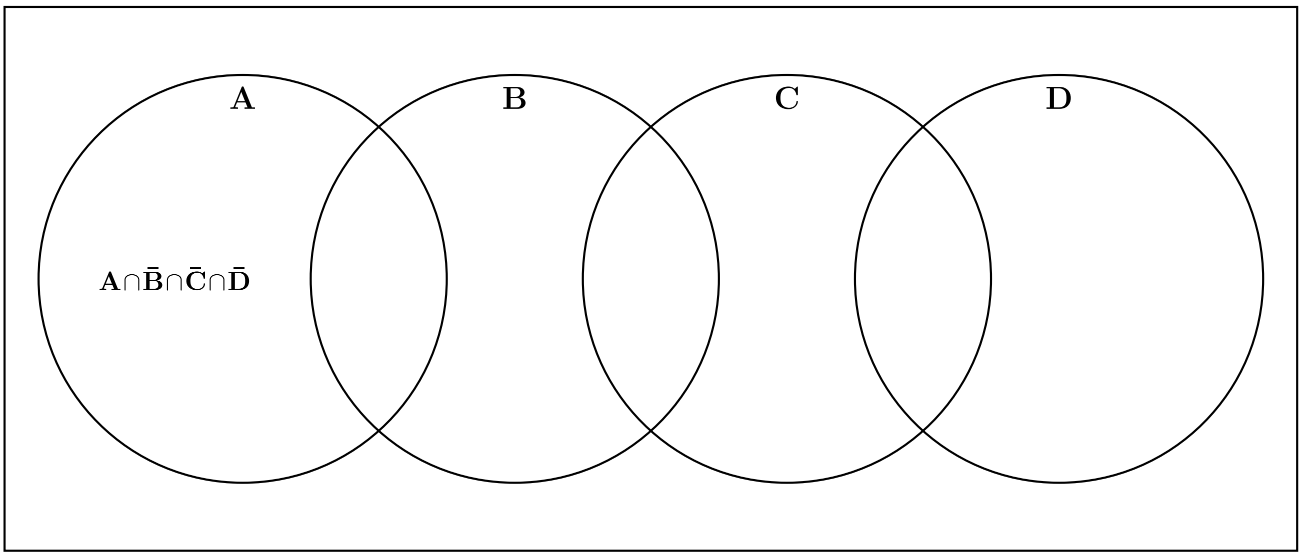
2. The provided Venn diagram illustrates four sets: $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$. A full Venn diagram of four sets defines 16 distinct zones based on set membership. Your goal is to identify which of these 16 potential zones are represented in this specific diagram. For each zone present, clearly indicate its corresponding location in the figure.

   The 16 possible zones are denoted by:

$$\begin{array}{llll} \mathbf{A}\cap\mathbf{B}\cap\mathbf{C}\cap\mathbf{D} & \mathbf{A}\cap\mathbf{B}\cap\mathbf{C}\cap\bar{\mathbf{D}} & \mathbf{A}\cap\mathbf{B}\cap\bar{\mathbf{C}}\cap\mathbf{D} & \mathbf{A}\cap\mathbf{B}\cap\bar{\mathbf{C}}\cap\bar{\mathbf{D}} \\ \mathbf{A}\cap\bar{\mathbf{B}}\cap\mathbf{C}\cap\mathbf{D} & \mathbf{A}\cap\bar{\mathbf{B}}\cap\mathbf{C}\cap\bar{\mathbf{D}} & \mathbf{A}\cap\bar{\mathbf{B}}\cap\bar{\mathbf{C}}\cap\mathbf{D} & \mathbf{A}\cap\bar{\mathbf{B}}\cap\bar{\mathbf{C}}\cap\bar{\mathbf{D}} \\ \bar{\mathbf{A}}\cap\mathbf{B}\cap\mathbf{C}\cap\mathbf{D} & \bar{\mathbf{A}}\cap\mathbf{B}\cap\mathbf{C}\cap\bar{\mathbf{D}} & \bar{\mathbf{A}}\cap\mathbf{B}\cap\bar{\mathbf{C}}\cap\mathbf{D} & \bar{\mathbf{A}}\cap\mathbf{B}\cap\bar{\mathbf{C}}\cap\bar{\mathbf{D}} \\ \bar{\mathbf{A}}\cap\bar{\mathbf{B}}\cap\mathbf{C}\cap\mathbf{D} & \bar{\mathbf{A}}\cap\bar{\mathbf{B}}\cap\mathbf{C}\cap\bar{\mathbf{D}} & \bar{\mathbf{A}}\cap\bar{\mathbf{B}}\cap\bar{\mathbf{C}}\cap\mathbf{D} & \bar{\mathbf{A}}\cap\bar{\mathbf{B}}\cap\bar{\mathbf{C}}\cap\bar{\mathbf{D}} \end{array}$$

   Examples: The zone $\mathbf{A}\cap\bar{\mathbf{B}}\cap\bar{\mathbf{C}}\cap\bar{\mathbf{D}}$ is represented in the diagram because it corresponds to the region in the diagram containing objects that belong exclusively to set **A** and none of the other three sets. Conversely, the zone $\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}\cap\mathbf{D}$ is not represented in the diagram because there is no specific region in the diagram where all four sets intersect.

   a. Identify and mark all the zones represented in the diagram by labeling their corresponding regions.



   b. List all the zones that are not represented in the diagram.

3. At a sports festival, athletes could win medals in five different sports: Basketball, Tennis, Swimming, Running, and Cycling. Based on the following data, determine the total number of athletes who participated in the festival.
   - Only one athlete did not win a medal in any of the five sports.
   - Exactly 2 athletes won a medal in all five sports.
   - Exactly 3 athletes won medals in Basketball, Tennis, and Swimming but did not win a medal in Running or Cycling.
   - The rest of the athletes won a medal in exactly one sport.
   - The total counts of medals awarded for each sport were:
     - 12 medals in Basketball.
     - 10 medals in Tennis.
     - 8 medals in Swimming.
     - 6 medals in Running.
     - 4 medals in Cycling.

   Justify your answer.

4. Prove the following identity involving the four boolean variables x, y, z, and w:

$$(\mathbf{x} \wedge \mathbf{y}) \vee (\mathbf{y} \wedge \mathbf{z}) \vee (\mathbf{z} \wedge \mathbf{w}) \vee (\mathbf{w} \wedge \mathbf{x}) \equiv (\mathbf{x} \vee \mathbf{z}) \wedge (\mathbf{y} \vee \mathbf{w})$$

Remark: If you choose to use a truth table for your proof, you must use the provided four-variable skeleton tables on the following page, adding columns as necessary for your intermediate steps.

x	y	z	w	
T	T	T	T	
T	T	T	F	
T	T	F	T	
T	T	F	F	
T	F	T	T	
T	F	T	F	
T	F	F	T	
T	F	F	F	
F	T	T	T	
F	T	T	F	
F	T	F	T	
F	T	F	F	
F	F	T	T	
F	F	T	F	
F	F	F	T	
F	F	F	F	

x	y	z	w	
T	T	T	T	
T	T	T	F	
T	T	F	T	
T	T	F	F	
T	F	T	T	
T	F	T	F	
T	F	F	T	
T	F	F	F	
F	T	T	T	
F	T	T	F	
F	T	F	T	
F	T	F	F	
F	F	T	T	
F	F	T	F	
F	F	F	T	
F	F	F	F	

In a few sentences, provide a justification for why your truth tables confirm the validity of the identity.

5. For each of the following four logical propositions involving boolean variables x and y, determine whether the statement is a tautology.

- If the proposition is a tautology, provide a formal proof demonstrating that the proposition evaluates to true for all four possible truth assignments to x and y.
- If the proposition is not a tautology, identify at least one truth assignment for x and y (a counterexample) for which the proposition evaluates to false.

A reference truth table is provided below for the two-variable functions used in these propositions: $\mathcal{AND}$ ($\wedge$), $\mathcal{OR}$ ($\vee$), $\mathcal{NAND}$ ($\uparrow$), and $\mathcal{NOR}$ ($\downarrow$). Also provided below is the truth table of the $\mathcal{IMPLY}$ ($\rightarrow$) function of two propositions P and Q.

x	y	$x \wedge y$	$x \vee y$	$x \uparrow y$	$x \downarrow y$
T	T	T	T	F	F
T	F	F	T	T	F
F	T	F	T	T	F
F	F	F	F	T	T

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

a. $(x \vee y) \rightarrow (x \wedge y)$

b. $(x \vee y) \rightarrow (x \uparrow y)$

c. $(x \downarrow y) \rightarrow (x \wedge y)$

d. $(x \downarrow y) \rightarrow (x \uparrow y)$

6. A card contains the following statements regarding two distinct integers, n and k:

   - $n > k$.
   - $k > n$.
   - Exactly one of the three statements on this card (including this one) is true.

   Prove that this situation constitutes a paradox. Specifically, demonstrate that the third statement can be neither true nor false without creating a logical contradiction.