Discrete Math Quiz: Number Theory

Name and ID: .....

1. Find the prime factors of the following two numbers:
   (a) $252 =$ _____
   (b) $103 =$ _____

2. Compute $(n \bmod d)$ for the following n and d.
   (a) $(101 \bmod 3) =$ _____
   (b) $(101 \bmod 5) =$ _____
   (c) $(101 \bmod 7) =$ _____
   (d) $(101^2 \bmod 3) =$ _____
   (e) $(101^2 \bmod 5) =$ _____
   (f) $(101^2 \bmod 7) =$ _____

3. Find, if it exists, $(n^{-1} \bmod d)$ (inverse of n modulo d) for the following n and d.
   (a) $(3^{-1} \bmod 7) =$ _____
   (b) $(4^{-1} \bmod 7) =$ _____
   (c) $(5^{-1} \bmod 6) =$ _____
   (d) $(3^{-1} \bmod 6) =$ _____

4. Compute $\varphi(n)$ for the following n.
   (a) $\varphi(17) =$ _____
   (b) $\varphi(25) =$ _____
   (c) $\varphi(35) =$ _____
   (d) $\varphi(54) =$ _____

5. Compute $(n^k \bmod d)$ for the following n, k, and d.
   (a) $(2^{200} \bmod 3) =$ _____
   (b) $(100^{16} \bmod 17) =$ _____
   (c) $(1001^8 \bmod 15) =$ _____

6. Find the greatest common divisors for the following set of numbers.
   (a) $\gcd(64, 81) =$ _____
   (b) $\gcd(18, 27, 45, 63) =$ _____

7. Find the least common multiply in the first part and answer the question in the second part.
   (a) $\operatorname{lcm}(18, 27, 45) =$ _____
   (b) What is the smallest integer $n > 3$ for which $(n \bmod 6) = (n \bmod 8) = 3$? _____

8. Bonus: Compute $10! \bmod 11 =$ _____