Solutions to Discrete Structures TR11 Number Bases Quiz

1. Write the decimal number $(40)_{10}$ in its binary (base-2) and its ternary (base-3) representations.

$$\begin{aligned} \mathbf{40} = 32 + 8 = 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 &\implies (40)_{10} = (\mathbf{101000})_{\mathbf{2}} \\ \mathbf{40} = 27 + 9 + 3 + 1 = 1 \cdot 3^3 + 1 \cdot 3^2 + 1 \cdot 3^1 + 1 \cdot 3^0 &\implies (40)_{10} = (\mathbf{1111})_{\mathbf{3}} \end{aligned}$$

2. Write the twelve decimal numbers $(1)_{10}, (2)_{10}, \ldots, (12)_{10}$ in their base-6 representation.

$$\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}, \mathbf{10}, \mathbf{11}, \mathbf{12}, \mathbf{13}, \mathbf{14}, \mathbf{15}, \mathbf{20}$$

3. Which number is larger $(212)_4$ or $(100110)_2$?

$$\begin{aligned} (\mathbf{212})_{\mathbf{4}} &= 2 \cdot 4^2 + 1 \cdot 4^1 + 2 \cdot 4^0 = 32 + 4 + 2 = \mathbf{38} \\ (\mathbf{100110})_{\mathbf{2}} &= 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 32 + 4 + 2 = \mathbf{38} \end{aligned}$$

4. In the base-14 system: $A = 10$, $B = 11$, $C = 12$ and $D = 13$. What is the decimal value of the number $(BAD)_{14}$?

$$(\mathbf{BAD})_{\mathbf{14}} = 11 \cdot 14^2 + 10 \cdot 14^1 + 13 \cdot 14^0 = 2156 + 140 + 13 = (\mathbf{2309})_{\mathbf{10}}$$

5. Characterize **all** the base-2 numbers that are multiples of 2 (those numbers that are divisible by 2)?

   Answer: A base-2 number x is a multiple of 2 **if and only if** its last digit is 0.

   Generalization: Which base-b numbers are multiples of b?

   Answer: A base-b number x is a multiple of b **if and only if** its last digit is 0. This is true because all the other digits of x add multiples of b^i for $i \geq 1$ and therefore add multiples of b to the value of x.

6. What happens to a base-3 number when a 1 is added after the last (least significant) digit?
   Examples: 220 becomes 2201 and 1212 becomes 12121.

   Answer: Let x be a base-3 number, then

$$(\mathbf{x1})_{\mathbf{3}} = \mathbf{3x + 1}$$

   Generalization: What happens to a base-b number when a $0 \leq c < b$ is appended at its end?

   Answer: Let x be a base-b number, then

$$(\mathbf{xc})_{\mathbf{b}} = \mathbf{bx + c}$$