Solutions to Discrete Structures TR2 Number Systems Quiz

1. Write the decimal number $(63)_{10}$ in its binary (base-2) and its ternary (base-3) representations.

$$63 = 32 + 16 + 8 + 4 + 2 + 1 = 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 \implies (63)_{10} = (111111)_2$$
$$63 = 2 \cdot 27 + 9 = 2 \cdot 3^3 + 1 \cdot 3^2 + 0 \cdot 3^1 + 0 \cdot 3^0 \implies (63)_{10} = (2100)_3$$

2. Write the twelve decimal numbers $(1)_{10}, (2)_{10}, \ldots, (12)_{10}$ in their base-5 representation.

$$1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, 22$$

3. Which number is larger $(222)_4$ or $(101010)_2$?

$$
\begin{aligned}
(222)_4 &= 2 \cdot 4^2 + 2 \cdot 4^1 + 2 \cdot 4^0 = 32 + 8 + 2 = 42 \\
(101010)_2 &= 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 32 + 8 + 2 = 42
\end{aligned}
$$

4. In the base-15 system: $A = 10$, $B = 11$, $C = 12$, $D = 13$, and $E = 14$. What is the decimal value of the number $(ACE)_{15}$?

$$(ACE)_{15} = 10 \cdot 15^2 + 12 \cdot 15^1 + 14 \cdot 15^0 = 2250 + 180 + 14 = (2444)_{10}$$

5. Characterize **all** the base-3 numbers that are multiples of 3 (those numbers that are divisible by 3)?

   Answer: A base-3 number x is a multiple of 3 **if and only if** its last digit is 0.

   Generalization: Which base-b numbers are multiples of b?

   Answer: A base-b number x is a multiple of b **if and only if** its last digit is 0. This is true because all the other digits of x add multiples of b^i for $i \geq 1$ and therefore add multiples of b to the value of x.

6. What happens to a binary number when a 1 is added after the last (least significant) digit? Examples: 110 becomes 1101 and 1011 becomes 10111

   Answer: Let x be a binary number, then

$$(x1)_2 = 2x + 1$$

   Generalization: What happens to a base-b number when a $0 \leq c < b$ is appended at its end?

   Answer: Let x be a base-b number, then

$$(xc)_b = bx + c$$